Univalent Relational Gauge Dynamics: A Constructive Groupoid Field Theory of Identity Transport, Curvature, Defect Matter, and Emergent Geometry

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Modern gauge theory already treats many field configurations as different presentations of one physical situation, but it normally begins with a chosen gauge group, spacetime complex, field content, and action. This paper develops the first constructive model of Univalent Relational Dynamics by reversing that order. A physical configuration is defined on a finite oriented 2-complex as local identity objects at vertices, admissible equivalences along edges, and coherence data around faces. Local automorphisms act as gauge transformations; curvature is the failure of identity transport to close; stable nontrivial holonomy sectors behave as defects; and their minimum invariant coherence cost defines a candidate mass. A conjugation-invariant coherence action is introduced and shown to reduce, in the one-object groupoid limit, to ordinary lattice gauge structure. Quantization is formulated as a finite sum over equivalence classes weighted by automorphism volume. The model is designed to separate established mathematics from new physical hypotheses and to produce exact computational tests. Its decisive objective is not to rename gauge theory, but to determine whether gauge symmetry, defect charge, mass gaps, effective geometry, and observable truncation can arise from one equivalence-first dynamical law.

Keywords: Univalent Relational Dynamics, equivalence-first physics, groupoid field theory, lattice gauge theory, category theory, homotopy type theory, univalence, higher gauge theory, cohesive homotopy type theory, identity transport, coherence action, gauge automorphisms, holonomy, curvature, topological defects, emergent matter, mass gap, homotopy quotient, groupoid quantization, emergent geometry, quantum gravity, GenesisBoard, Theory of Everything.

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Outline

1. From an Equivalence-First Ontology to a Constructive Theory

1.1 The achievement and limitation of Reality as Equivalence

Summarize the existing framework: states are presentations, admissible equivalence constitutes physical identity, higher coherence has physical content, observables descend through equivalence, and dynamics governs the formation and obstruction of equivalence.

Identify the present limitation. These statements define a research program but do not yet specify one complete field theory whose configurations, action, observables, partition function, limiting cases, and predictions can be calculated.

1.2 The objective of the present paper

Construct a minimal finite model in which:

local identity gives gauge symmetry,

transport gives connection,

failed closure gives curvature,

stable obstruction gives matter,

minimum coherence cost gives mass,

and summation over equivalence classes gives quantization.

1.3 Established mathematics versus new physics

Separate three layers explicitly:

established groupoid, gauge, lattice, and homotopy mathematics;

the new Univalent Relational interpretation;

new dynamical hypotheses that can succeed or fail.

1.4 Standard of success

The model must accomplish four things:

recover ordinary lattice gauge structure as a special case;

produce an exactly computable finite system;

derive at least one result not inserted by definition;

state conditions that would falsify the proposed physical interpretations.

2. The Mathematical Arena

2.1 Finite oriented 2-complex

Let K consist of:

vertices V,

oriented edges E,

faces F,

source and target maps s,t : E → V,

and an ordered boundary ∂f for every face f.

The 2-complex is initially a computational scaffold, not assumed to be fundamental spacetime.

2.2 The groupoid of local identity

Let G be a finite or compact groupoid.

Its objects represent admissible local identity types. Its morphisms represent reversible physical identifications between local presentations.

For each object A, the self-equivalences form the automorphism group Aut(A).

2.3 Field configurations

A configuration assigns:

an identity object A_v to every vertex v;

an invertible transport morphism U_e : A_s(e) → A_t(e) to every oriented edge e;

optional local matter data ψ_v in a representation associated with A_v.

Orientation reversal requires:

U_e^-1 = U_reverse(e).

2.4 Why a groupoid rather than a single group

A one-object groupoid reproduces conventional gauge theory with one gauge group G = Aut(A).

A many-object groupoid permits:

different local presentations,

equivalences among distinct presentations,

different automorphism sectors,

dynamical transitions among identity types,

and possible selection of an effective gauge group.

2.5 Higher extension

Introduce 2-morphisms only after the ordinary groupoid model has been solved. These will compare alternative transports and provide the next level of gauge-of-gauge coherence.

3. Physical Equivalence and Gauge Freedom

3.1 Local changes of presentation

At each vertex choose:

$g_v \in Aut(A_v)$.

The edge transport transforms as:

$U_e' = g_{t(e)} \circ U_e \circ g_{s(e)}^{-1}$.

This is interpreted as a change of local presentation rather than a physical change.

3.2 Gauge orbit and physical configuration

Two assignments $\{A_v, U_e\}$ belong to the same physical configuration when related by the family $\{g_v\}$.

The physical configuration space is therefore a groupoid or homotopy quotient, not merely a set quotient.

3.3 Admissibility

Not every formal equivalence is automatically physical. Define an admissibility selector:

$E_{phys}(A,B) \subseteq Hom_G(A,B)$.

The selector must preserve:

composition,

inverses,

relevant observables,

boundary conditions,

and the chosen physical structures.

3.4 Observable descent

An observable O is physical only when:

$O(A,U) = O(A,U')$

for gauge-related configurations, or when the two outputs are canonically equivalent in the observable category.

3.5 First theorem target

Prove that conjugacy-invariant functions of face holonomy descend to the physical quotient.

4. Identity Transport, Holonomy, and Curvature

4.1 Path transport

For a path $p = e_n \dots e_2 e_1$, define:

$U_p = U_{en} \circ \dots \circ U_{e2} \circ U_{e1}$.

This transports identity from the source of the path to its target.

4.2 Face holonomy

For a closed oriented face boundary based at v, define:

$H_f = U_{en} \circ \dots \circ U_{e1} \in Aut(A_v)$.

4.3 Transformation of holonomy

Under local gauge transformations:

$H_f' = g_v \circ H_f \circ g_v^{-1}$.

Its conjugacy class is therefore presentation-independent.

4.4 Curvature as failed identity closure

The face is flat when:

$H_f = id_{Av}$.

Nontrivial H_f measures the failure of transported identity to return trivially after traversing the face.

This gives a precise finite meaning to the phrase:

curvature is failed identity closure.

4.5 Higher coherence

When several decompositions of a surface yield transports that are not strictly equal, introduce 2-morphisms relating them. Their consistency conditions become higher curvature and higher obstruction data.

5. The Univalent Relational Coherence Action

5.1 Requirements on the action

The action must be:

gauge invariant,

local on the finite complex,

bounded below in its Euclidean form,

computable,

sensitive to nontrivial holonomy,

and capable of supporting stable defect sectors.

5.2 Face coherence term

Choose a nonnegative conjugacy-invariant function K such that:

K(id) = 0.

Define:

$S_{face} = \beta \Sigma_f w_f K(H_f)$.

Here w_f is a face weight and β controls the cost of failed closure.

5.3 Transport and matter term

For represented local data ψ_v, define a covariant edge difference:

$D_e \psi = \psi_{t(e)} - \rho(U_e)\psi_{s(e)}$.

Then:

$S_{matter} = \kappa \Sigma_e ||D_e \psi||^2 + \Sigma_v V(\psi_v)$.

5.4 Identity-selection term

If several object types A are permitted, introduce:

$S_{identity} = \Sigma_v M(A_v)$,

where M does not represent ordinary particle mass at this stage. It is a local cost assigned to maintaining a particular identity structure.

5.5 Defect and boundary terms

Add terms that distinguish:

nontrivial topological sectors,

boundary conditions,

global gluing obstruction,

and localized failure of trivialization.

5.6 Complete finite action

The initial Euclidean action is:

$$S_{URD} = \beta \Sigma_f w_f K(H_f) + \kappa \Sigma_e ||D_e \psi||^2 + \Sigma_v V(\psi_v) + \Sigma_v M(A_v) + S_{boundary} + S_{defect}.$$

5.7 Principle of maximal coherent equivalence

Low-action configurations preserve the greatest admissible coherence compatible with boundary conditions, topology, and stable physical distinction.

The principle does not demand universal triviality. Nontrivial stable sectors may survive because topology or boundary data prevents complete coherent closure.

6. Recovery of Ordinary Lattice Gauge Theory

6.1 One-object reduction

Restrict G to one object A.

Then every edge morphism belongs to:

$Aut(A) = G$.

The model becomes a conventional G-valued lattice transport theory.

6.2 Wilson-type face action

For a unitary representation ρ, choose:

$K(H_f) = \mathrm{Re}\,\mathrm{Tr}[I - \rho(H_f)]$.

The face action becomes the Wilson plaquette form up to normalization and lattice weights.

6.3 Recovery theorem

Prove:

When the identity groupoid has one object, all edge transports are automorphisms of that object, and K is chosen as the Wilson class function, the Univalent Relational coherence action reduces to ordinary lattice gauge theory.

6.4 Interpretation of the theorem

This result would establish that conventional gauge theory is a restricted phase of the equivalence-first model:

one identity type,

one automorphism group,

fixed lattice,

no transitions among object types,

and no higher morphism dynamics.

6.5 What is not yet derived

The theorem would not derive the Standard Model gauge group. It would show how any ordinary gauge theory appears inside the more general structure.

Gauge-group selection requires a separate dynamical result.

7. Defect Matter, Charge, and Mass

7.1 Defects as obstruction classes

Define a defect as a localized or global obstruction to trivializing the transport assignment.

Possible invariants include:

nontrivial face holonomy,

conjugacy class,

winding data,

cohomological obstruction,

or failure of compatible global gluing.

7.2 Charge as automorphism response

A defect sector D carries charge through a representation:

$\rho_D : Aut(A) \to GL(V_D)$.

Charge is therefore the response of a persistent identity pattern to its local self-equivalences.

7.3 Mass as invariant coherence cost

For a static defect class D, define:

$m_D c^2 = \inf S_{static}[A,U,\psi]$

over all presentations belonging to D and satisfying the same physical boundary conditions.

This minimization is essential. It removes arbitrary descriptive complexity and retains only the least cost required to sustain the defect.

7.4 Stability

A defect represents particle-like matter only when it is:

localized,

persistent under admissible perturbation,

protected against continuous removal,

and associated with a finite positive minimum cost.

7.5 Mass gap

Define the finite-model mass gap as the difference between the vacuum minimum and the lowest nontrivial defect minimum.

The model should determine whether this gap remains nonzero under increasing complex size and controlled scaling.

7.6 Failure condition

If the proposed mass depends on gauge representative, arbitrary triangulation, or irrelevant presentation detail, then mass as equivalence inertia is rejected in this model.

8. Quantization Over the Moduli Groupoid

8.1 Finite Euclidean partition function

For a finite groupoid and finite complex, define:

$Z = \Sigma_{[\Phi]} \exp[-S_{URD}(\Phi)/\hbar] / |Aut(\Phi)|$,

where $[\Phi]$ runs over physical equivalence classes and $Aut(\Phi)$ is the stabilizer of the configuration.

8.2 Why automorphism weighting matters

Configurations with large self-symmetry must not be overcounted merely because they possess many equivalent presentations.

The automorphism factor implements a finite groupoid-cardinality correction.

8.3 Observables

For a gauge-invariant observable O:

$= Z^{-1} \Sigma_{[\Phi]} O(\Phi) \exp[-S_{URD}(\Phi)/\hbar] / |Aut(\Phi)|$.

8.4 Real-time amplitude

After the finite Euclidean model is understood, define a real-time analogue using $\exp[iS/\hbar]$.

Do not begin with the real-time form because oscillatory sums can obscure the basic mathematical tests.

8.5 Interference among identifications

Distinct admissible transport structures connecting the same boundary data contribute separate amplitudes.

This gives an exact finite test of the proposal that quantum interference may be interpreted as interference among coherent identifications.

8.6 Measurement as truncation

Construct an explicit map from the configuration groupoid to its set of connected outcome classes:

$\tau_o : M \rightarrow \pi_o(M)$.

Determine what probabilistic and coherence information is lost under this map.

9. Dynamical Selection of Gauge Structure

9.1 The central problem

If the automorphism group is fixed in advance, the theory recovers gauge dynamics but does not explain why a particular gauge symmetry exists.

9.2 Heterogeneous identity groupoid

Permit several object types:

A_1, A_2, ..., A_n,

with different automorphism groups and different spaces of admissible equivalence.

9.3 Competition among sectors

Evaluate the free energy of each sector:

F_i = -k_B T log Z_i,

or its zero-temperature action minimum.

Test whether coherence, entropy, defect stability, and boundary compatibility favor one effective automorphism sector.

9.4 Spontaneous gauge selection

A gauge structure is dynamically selected when a sector dominates without its group being imposed as the unique starting choice.

9.5 Symmetry breaking

Transitions between object types or reductions of automorphism groups provide a groupoid-level model of symmetry breaking.

9.6 Non-inserted result

The first genuinely new result should be one of the following:

selection of a preferred automorphism sector;

a quantized defect spectrum;

a nonzero mass gap;

a universal relationship between defect charge and minimum coherence cost;

or a restricted class of stable holonomies.

10. Emergent Geometry and the Gravity Bridge

10.1 Relational distance

Define a candidate distance between vertices or identity contexts:

$d(v,w) = \inf_p C(U_p)$,

where the infimum is over admissible paths connecting v and w.

10.2 Conditions for a metric

Test:

positivity,

separation,

symmetry or directed asymmetry,

and the triangle inequality.

Failure of these conditions may indicate a generalized, causal, or asymmetric geometry rather than an ordinary metric space.

10.3 Effective dimension

Estimate dimension from:

ball-volume growth,

spectral dimension,

diffusion return probability,

connectivity scaling,

and the behavior of low-cost equivalence neighborhoods.

10.4 Curvature comparison

Compare holonomy-derived curvature with graph, Regge, or continuum curvature in controlled complexes.

10.5 Entropy of equivalence fibers

For a coarse physical meaning m, define:

$S_{eq}(m) = k_B \log \mathrm{Vol}[q^{-1}(m)]$,

with automorphism-aware counting.

10.6 Thermodynamic gravity conjecture

Investigate whether variation of equivalence-fiber entropy across an effective causal boundary can satisfy:

$\delta Q = T\, dS_{eq}$.

Only after establishing this relation should the model attempt to connect with an Einstein-type equation of state.

10.7 Status

The gauge recovery is intended as the first demonstrated result. Gravity remains a second-stage continuum conjecture unless an explicit derivation is obtained.

11. GenesisBoard Implementation and Computational Experiments

11.1 Extension of GenesisBoard

Add explicit support for:

vertex identity types,

vertex automorphism groups,

oriented transport morphisms,

face boundaries,

holonomy,

gauge transformations,

defect classification,

and exact quotient counting.

11.2 Baseline experiments

Run:

random transport controls,

flat-coherence controls,

single-object gauge models,

heterogeneous groupoid models,

defect-seeded and defect-free models.

11.3 Primary observables

Measure:

total coherence action,

face-holonomy distribution,

gauge-orbit size,

automorphism-weighted state count,

defect density,

defect lifetime,

mass-gap estimate,

correlation length,

spectral dimension,

and sector free energy.

11.4 Gauge-invariance test

Apply randomly generated local changes of presentation and verify numerically that:

the action,

partition function,

defect class,

and physical observables

remain unchanged.

11.5 Scaling tests

Increase complex size and compare:

finite-size behavior,

continuum tendencies,

mass-gap stability,

defect persistence,

and effective-dimensional convergence.

11.6 Reproducibility

Publish:

source code,

random seeds,

complete parameter files,

raw outputs,

analysis scripts,

null models,

and predefined acceptance criteria.

12. Predictions, Falsification, and the Path Toward a Theory of Everything

12.1 Minimal success

The paper succeeds as a constructive theory if it:

defines the model rigorously;

proves gauge invariance;

recovers lattice gauge theory in the one-object limit;

and computes the complete finite partition function for at least one nontrivial example.

12.2 Strong success

The paper makes a substantive physical advance if it also derives a result not inserted by hand, such as a preferred sector, quantized stable defect, or nonzero scaling mass gap.

12.3 Falsification criteria

Reject or substantially revise the model if:

no physically meaningful admissibility selector can be defined;

the coherence action fails gauge invariance;

ordinary gauge theory cannot be recovered cleanly;

defect mass is presentation-dependent;

the finite quantum sum is inconsistent or divergent in its intended domain;

or all outputs merely reproduce assumptions without additional constraint.

12.4 Relation to a Theory of Everything

This model does not claim to be a completed Theory of Everything.

It is the proposed finite kernel from which a candidate ToE would still have to derive:

the observed gauge group,

fermionic and chiral matter,

three generations,

the Higgs sector,

particle masses and mixing,

Lorentzian 3+1-dimensional geometry,

general relativity,

quantum probabilities,

and experimentally distinguishable predictions.

12.5 Research sequence

The recommended progression is:

finite groupoid gauge dynamics;

defect spectrum and mass gap;

higher-groupoid extension;

continuum gauge-field limit;

quantum measurement and probability;

emergent geometry;

thermodynamic gravity;

Standard Model selection;

phenomenology.

12.6 Final conclusion

The decisive question is no longer whether category theory and homotopy type theory can describe physics. They can.

The decisive question is whether a single dynamics of admissible equivalence can explain why particular gauge structures, particles, masses, geometries, and observable outcomes occur.

A successful finite model would be the first working machine of Univalent Relational Dynamics and the necessary bridge between an equivalence-first ontology and a genuine candidate Theory of Everything.

References

Art

1. From an Equivalence-First Ontology to a Constructive Theory

1.1 The Achievement and Limitation of Reality as Equivalence

The preceding framework, developed in Reality as Equivalence, proposed an inversion of the conventional explanatory order of physics. Standard physical theories generally begin with a space of states, fields, configurations, or histories. Equivalence is then imposed afterward through gauge freedom, coordinate invariance, symmetry, basis change, redundancy, or quotient construction. The equivalence-first proposal reverses this order. It treats an observed state as a local presentation and assigns physical identity to the structured family of admissible transformations that connects that presentation to others.

This reversal changes the status of equivalence. Equivalence is no longer merely a relation used to remove redundant descriptions from a previously defined state space. It becomes part of the constitution of the physical object. A particle, field, geometry, or composite system is identified through the transformations under which it persists, the automorphisms it admits, the processes through which it can be transported, and the higher coherence relations that make those transformations mutually compatible.

The central statement may be expressed as:

physical identity = presentation + admissible equivalence + coherence.

A presentation alone does not determine what is physically real because it may contain arbitrary coordinate, gauge, basis, or representational choices. An equivalence relation alone is also insufficient because it records only whether two presentations are associated, not how they are associated, how many distinct transformations connect them, which self-transformations preserve them, or whether different transformations are themselves related by higher transformations. The relevant physical object is therefore not merely an equivalence class. It is a structured moduli object containing presentations, transformations, automorphisms, loops, higher paths, and obstructions.

This leads naturally from sets to groupoids, from groupoids to higher groupoids, and from ordinary equality to identity types. Category theory supplies the language of composable processes. Higher category theory supplies transformations between transformations. Homotopy type theory interprets identity as a path and higher identity as a path between paths. Univalence provides the formal principle that structurally equivalent objects may be identified within the foundational language.

Reality as Equivalence used this architecture to propose a unified interpretation of several major physical concepts. Gauge symmetry was treated as local self-equivalence. Curvature was interpreted as failure of transported identity to close trivially around a loop. Charge was

associated with the response of a physical identity pattern under its automorphism structure. Matter was interpreted as stable coherence or obstruction. Mass was proposed as the cost of preserving and transporting such coherence. Quantum superposition was reinterpreted as unresolved higher equivalence structure. Entanglement was defined as failure of joint identity to factor into independent identities. Measurement was modeled as truncation of a higher relational object to a lower level of accessible distinction. Entropy was associated with the multiplicity and topology of presentations realizing the same coarse physical meaning. Spacetime was proposed to emerge from the geometry of coherent transport.

These proposals form a unified conceptual system. Their strength lies in the fact that they do not require separate primitive explanations for gauge symmetry, matter, geometry, quantum coherence, and observation. Each is treated as a different manifestation of one underlying relational architecture. The common elements are equivalence, transport, composition, coherence, obstruction, and truncation.

However, conceptual unity is not yet constructive physical theory.

The previous framework did not specify one complete mathematical model in which all of its central claims could be tested simultaneously. It introduced a schematic coherence action, but it did not fully define the configuration space over which that action operates. It proposed mass as equivalence inertia, but it did not calculate a defect spectrum or produce a numerical mass ratio. It proposed charge as automorphism response, but it did not derive a specific gauge group or representation content. It described measurement as truncation, but it did not derive probabilities or recover the Born rule. It proposed emergent geometry, but it did not establish a continuum limit producing a Lorentzian metric or Einstein-like dynamics.

This distinction separates an ontological framework from a physical model.

An ontological framework identifies the kinds of entities, transformations, and relations that should be regarded as fundamental. A physical model must additionally state:

which mathematical objects represent configurations;

which transformations count as admissible equivalences;

which quantities are observable;

which action or update law determines dynamics;

which boundary conditions define the problem;

which equations follow from variation or optimization;

which limits reproduce established physics;

and which predictions distinguish the model from existing theories.

Without these elements, a framework can organize physical thought but cannot yet generate experimentally assessable results.

The present paper therefore begins from the achievements of the equivalence-first ontology while addressing its main limitation. It seeks to replace a collection of broad correspondences with one finite constructive model in which the equivalence structure is explicit, the action is computable, gauge invariance can be proved, defect sectors can be classified, and the partition function can be evaluated.

The goal is not to abandon the wider ambitions of Univalent Relational Dynamics. It is to give those ambitions a controlled mathematical starting point.

1.2 The Objective of the Present Paper

The objective of this paper is to construct a minimal field theory in which local identity, transport, curvature, defect matter, charge, mass, and quantization arise within one equivalence-first structure.

The model begins with a finite oriented 2-complex. The vertices represent local physical contexts or local presentations. The oriented edges represent admissible transport between neighboring contexts. The faces represent elementary loops around which coherence can be tested. This finite setting is chosen because it permits exact definitions, direct numerical implementation, controlled comparison with lattice gauge theory, and explicit calculation of physical equivalence classes.

At each vertex v, the model assigns a local identity object A_v. This object is not interpreted as an isolated particle or state. It represents the local type of structure that is being transported and compared.

To each oriented edge e from v to w, the model assigns an invertible transport morphism:

U_e : A_v -> A_w.

The inverse orientation carries the inverse morphism:

U_reverse(e) = U_e^-1.

The transport morphism records how the local identity structure at one vertex is coherently identified with the identity structure at the next. It is the finite analogue of a connection or parallel-transport operator.

For a path p consisting of consecutive edges e_1, e_2, ..., e_n, the total transport is the composite:

U_p = U_en o ... o U_e2 o U_e1.

For a closed path around a face f, the composite becomes an automorphism of the starting identity object:

H_f = U_en o ... o U_e1.

The element H_f is the face holonomy. When H_f equals the identity automorphism, transport around the face closes coherently. When H_f differs from the identity, the loop possesses nontrivial curvature or obstruction.

This gives a precise finite formulation of the statement:

curvature is failed identity closure.

A local change of presentation is represented by an automorphism g_v of the identity object A_v at each vertex. Under such a change, the edge transport transforms as:

U_e -> g_t(e) o U_e o g_s(e)^-1.

The holonomy around a face transforms by conjugation. Any conjugacy-invariant function of H_f is therefore independent of the local presentation. This is the finite gauge principle of the model.

The paper then introduces a coherence action. Its simplest form assigns a nonnegative cost to nontrivial face holonomy:

S_face = beta sum_f w_f K(H_f),

where K is conjugacy-invariant and satisfies:

K(id) = 0.

The action may also include transported matter data, local identity costs, boundary conditions, and defect terms. The total action takes the schematic form:

S_URD = S_face + S_matter + S_identity + S_boundary + S_defect.

The central physical hypothesis is that realized structures favor coherent equivalence while remaining compatible with topological constraints, stable distinctions, and boundary data. Low-action configurations maximize coherent transport without necessarily collapsing the entire system into trivial uniformity.

The present model is designed to establish several specific results.

First, it will show that ordinary lattice gauge theory arises as a special case. When the identity groupoid contains only one object A, every edge transport belongs to the automorphism group:

G = Aut(A).

The model then reduces to a conventional G-valued lattice gauge theory. With an appropriate choice of K, the face-coherence action becomes the Wilson plaquette action.

Second, it will define defect sectors as obstructions to globally trivial transport. Such defects may be classified by holonomy, conjugacy class, winding data, cohomological information, or failure of global gluing.

Third, it will define charge as the representation response of a defect or excitation under local automorphisms. If a defect sector D transforms according to:

rho_D : Aut(A) -> GL(V_D),

then its charge is encoded in that representation.

Fourth, it will define mass as a minimum invariant coherence cost. For a defect class D, the proposed rest energy is:

m_D c^2 = inf S_static,

where the infimum is taken over all presentations belonging to the same physical defect class and satisfying the same physical boundary conditions.

Fifth, it will formulate quantization as a finite sum over physical equivalence classes rather than over arbitrarily repeated presentations:

Z = sum_[Phi] exp[-S_URD(Phi)/hbar] / |Aut(Phi)|.

The automorphism weighting prevents highly symmetric configurations from being overcounted merely because they possess many equivalent descriptions.

The immediate aim is therefore modest but decisive. The paper does not attempt to derive all known physics at once. It seeks to demonstrate that the equivalence-first ontology can generate a rigorous, finite, gauge-invariant, quantizable field theory with identifiable defect sectors and computable observables.

If successful, this construction will be the first working model of Univalent Relational Dynamics.

1.3 Established Mathematics Versus New Physics

A clear distinction must be maintained between established mathematical structure, interpretive reorganization, and genuinely new physical hypotheses.

The use of groups in gauge theory is established. The interpretation of a gauge field as a connection is established. Holonomy, curvature, principal bundles, lattice gauge theory, Wilson

loops, groupoids, higher groupoids, homotopy quotients, automorphism groups, and higher gauge transformations are established mathematical or physical concepts. Homotopy type theory and univalence are also established mathematical frameworks. None of these ingredients becomes novel merely by appearing in the present theory.

Similarly, the fact that a one-object groupoid corresponds to a group is standard. The transformation law:

U_e -> g_t(e) U_e g_s(e)^-1

is standard in lattice gauge theory. The use of conjugacy-invariant functions of plaquette holonomy is standard. The weighting of finite groupoid configurations by inverse automorphism order has precedents in groupoid cardinality and finite gauge theory.

The novelty of the present work does not lie in claiming ownership of these constructions.

The first new element is their ontological reordering. Conventional gauge theory usually begins with a selected gauge group acting on a selected field space over a selected spacetime. The present model begins instead with a groupoid of possible local identity structures and admissible equivalences. Gauge groups then appear as automorphism groups of those identity structures.

The distinction is important:

conventional order:

spacetime + gauge group + fields -> gauge equivalence;

equivalence-first order:

identity objects + admissible equivalence + coherence dynamics -> effective gauge structure, fields, and geometry.

The second new element is the interpretation of the action as a physical cost of maintaining coherent identity transport. A Wilson-type action may be recovered mathematically, but its role is reinterpreted. The face term does not merely measure field strength. It measures the failure of local identity transport to close.

The third new element is the proposal that stable defect energy defines mass as equivalence inertia. The topological classification of defects is well established in many theories. The novel proposal is that the least presentation-independent cost required to sustain a persistent coherence obstruction is the physical rest mass of the corresponding excitation.

The fourth new element is the possibility of a heterogeneous identity groupoid. Instead of fixing a single object with one automorphism group, the model may permit several object types with

different automorphism structures and transformation spaces. Their relative stability can then be evaluated dynamically. This opens the possibility that an effective gauge group could be selected rather than postulated.

The fifth new element is the use of one common structure to connect gauge symmetry, defect matter, charge, mass, quantization, measurement, and emergent geometry. Many of these concepts have independent precedents. The proposed novelty is their derivation from a shared equivalence-coherence architecture.

The sixth new element is the insistence that these ideas must be subjected to explicit failure tests. A poetic reinterpretation is insufficient. The model must show that its new physical definitions are invariant, calculable, and nontrivial.

Several statements in the theory must therefore initially be labeled as conjectures rather than conclusions.

It is a conjecture that particle-like matter can be identified with stable coherence defects in a physically realistic limit.

It is a conjecture that minimum defect coherence cost corresponds to observed rest mass.

It is a conjecture that automorphism-sector competition can select physically relevant gauge groups.

It is a conjecture that quantum amplitudes can be fundamentally interpreted as interference among admissible identifications.

It is a conjecture that classical measurement corresponds to an effective truncation of higher equivalence structure.

It is a conjecture that spacetime distance and gravitation can emerge from the cost and obstruction of identity transport.

The finite model is intended to determine which of these statements survive mathematical implementation.

The paper therefore follows a strict hierarchy of claims:

definitions specify the model;

propositions establish invariance and consistency;

theorems recover known special cases;

computations characterize finite behavior;

conjectures connect the finite model to fundamental physics;

predictions state what would differ from established theories.

This hierarchy is necessary for scientific credibility. It prevents established mathematical facts from being presented as new physical discoveries and prevents speculative physical interpretations from being mistaken for proven results.

1.4 Standard of Success

The model should be judged by explicit criteria rather than by the breadth of its conceptual reach.

The first requirement is mathematical completeness. The paper must specify the finite complex, identity groupoid, field configurations, admissible equivalences, gauge transformations, action, observables, boundary conditions, and quantization rule. Every symbol appearing in the model must have a defined domain and physical interpretation.

The second requirement is gauge invariance. The action and observables must be unchanged under local changes of presentation. This must be demonstrated algebraically and verified computationally.

The third requirement is recovery of a known theory. In the one-object groupoid limit, the model must reduce to ordinary lattice gauge structure. With the Wilson class function, the coherence action must reproduce the standard plaquette action up to explicit conventions and normalization.

This recovery result is essential. It shows that the theory contains established physics rather than replacing it with an unrelated formalism.

The fourth requirement is exact finite computability. For at least one nontrivial finite groupoid and finite 2-complex, the complete configuration space, gauge orbits, automorphism groups, action values, partition function, and selected observables should be calculable.

The fifth requirement is a non-inserted result. The model must produce at least one conclusion not defined into it at the beginning.

Examples include:

a preferred automorphism sector;

a stable quantized defect;

a finite mass gap;

a restricted holonomy spectrum;

a universal relation between charge and minimum defect cost;

or an emergent effective dimension.

The sixth requirement is presentation independence. A proposed physical quantity must not vary under arbitrary gauge choice, relabeling, triangulation artifact, or equivalent representation. This condition is especially important for mass as coherence cost. If the calculated mass depends on a descriptive choice, it is not yet a physical invariant.

The seventh requirement is scalability. Results obtained on small complexes must be examined under increasing system size, refinement, and variation of topology. A defect or mass gap that disappears immediately under scaling may be a finite artifact rather than a candidate physical structure.

The eighth requirement is falsifiability. The theory must state what outcomes would count against it.

The framework would require substantial revision if:

no coherent admissibility selector can distinguish physical equivalence from arbitrary mathematical equivalence;

the action is not invariant under local changes of presentation;

the one-object limit fails to reproduce ordinary gauge theory;

defect classes cannot be defined independently of presentation;

minimum defect cost does not stabilize under refinement;

different equivalent descriptions produce different observables;

the quantization rule overcounts gauge-equivalent configurations;

or every claimed result is merely a restatement of an input assumption.

The ninth requirement is disciplined scope. The present paper should not claim to derive the Standard Model, general relativity, quantum measurement, or cosmology unless those derivations are actually carried out. Its legitimate achievement would be narrower but important:

to show that equivalence-first physics can be formulated as a concrete gauge-invariant dynamical system.

The tenth requirement is reproducibility. All computational implementations should include source code, complete parameter definitions, random seeds, raw outputs, null models, and

acceptance criteria. A finite theory should be especially transparent because its principal results can, in principle, be reproduced exactly.

A successful outcome would justify the statement:

Univalent Relational Dynamics has been developed from an ontological proposal into a constructive candidate field theory.

A stronger success would occur if the model also produced a nontrivial defect spectrum, mass gap, or dynamically preferred symmetry sector.

Even that would not complete a Theory of Everything. It would, however, establish the indispensable first step: a working mathematical machine in which physical identity, gauge freedom, curvature, matter, and quantization arise from one common equivalence-coherence structure.

The present paper therefore asks a sharply limited question:

Can the equivalence-first ontology be converted into a finite, gauge-invariant, quantizable dynamics that recovers established gauge theory and generates additional physical structure?

The remainder of the paper constructs the model required to answer that question.

2. The Mathematical Arena

2.1 Finite Oriented 2-Complex

The constructive model begins with a finite oriented 2-complex K. This complex supplies the minimal combinatorial structure required to represent local contexts, transport between neighboring contexts, and coherence around elementary loops.

The 2-complex consists of three finite collections:

V = the set of vertices,

E = the set of oriented edges,

F = the set of oriented faces.

Each oriented edge e has a source and target:

s(e) in V,

t(e) in V.

The reversed orientation of e is denoted by e^-1 and satisfies:

s(e^-1) = t(e),

t(e^-1) = s(e).

Each face f is bounded by an ordered cyclic sequence of oriented edges:

partial f = (e_1, e_2, ..., e_n),

such that:

t(e_i) = s(e_(i+1))

for each consecutive pair, with the final edge returning to the starting vertex.

The face boundary therefore defines a closed path. This closure makes it possible to compare an identity structure with itself after transport around an elementary loop.

The complex K may be a triangular complex, square lattice, cubical 2-skeleton, irregular cellulation, or another finite decomposition. The formalism does not initially require a regular Euclidean lattice. This flexibility is important because the complex is not yet identified with physical spacetime.

In conventional lattice gauge theory, the lattice is normally regarded as a discretization of a previously defined manifold. Vertices approximate points, edges approximate short spacetime intervals, and faces approximate infinitesimal areas. The present construction adopts a more cautious interpretation.

The complex is initially a relational scaffold. It records:

which local contexts are adjacent;

which transports may be composed;

which closed loops can test coherence;

and which global topological structures may exist.

A vertex need not represent a point in a preexisting space. It may represent a local experimental context, algebraic presentation, computational state, coordinate patch, boundary condition, or local identity type. An edge records the possibility of coherent comparison or transport. A face records a minimal compositional relation among several such comparisons.

Geometry is therefore not assumed at the start. It may later be inferred from the costs, densities, and compositional properties of the transports assigned to K.

The finite 2-complex is the simplest structure that can support curvature. A graph containing only vertices and edges permits path transport but does not by itself specify elementary

surfaces around which loop closure can be evaluated. Faces provide the additional structure required to distinguish locally trivial transport from nontrivial holonomy.

This does not mean that two dimensions are physically fundamental. The term 2-complex indicates that cells up to dimension two are included. Such a complex may serve as the 2-skeleton of a higher-dimensional cell complex. Even in ordinary four-dimensional gauge theory, curvature is naturally tested on two-dimensional plaquettes or infinitesimal surfaces.

Higher cells may later impose relations among face holonomies. Three-cells can encode compatibility among surfaces, and higher cells can represent increasingly refined coherence conditions. The present paper begins at the 2-complex level because it is the first level at which gauge curvature, loop obstruction, and defect structure can be defined explicitly.

The orientation of edges and faces is essential.

An edge transport from v to w generally differs from transport from w to v, although the two are related by inversion when the transport is an equivalence. Likewise, reversing a face orientation reverses the order and direction of the edge transports around its boundary.

If a face f has boundary:

partial f = (e_1, e_2, ..., e_n),

then the oppositely oriented face f^-1 has boundary:

partial f^-1 = (e_n^-1, ..., e_2^-1, e_1^-1).

The corresponding holonomies will satisfy an inverse relationship.

The combinatorial data of K are therefore:

K = (V,E,F,s,t,partial).

Additional structures may be introduced as needed. These include:

edge lengths;

face areas;

vertex volumes;

incidence numbers;

boundary components;

preferred orientations;

causal directions;

or weights encoding physical scale.

In the minimal model, such quantities are not assumed to possess a fundamental geometric meaning. They are coefficients within the finite action. A continuum interpretation must be justified later through scaling behavior.

Boundary conditions also enter naturally at the level of the complex. Let:

partial K

denote the boundary subcomplex. Identity objects, transport morphisms, defect sectors, or observable data may be fixed or constrained on this boundary.

Different boundary conditions can define different physical questions.

For example:

fixed edge transport on the boundary may represent an imposed external gauge field;

fixed identity types may represent selected physical phases;

nontrivial boundary holonomy may force a defect into the interior;

periodic boundaries may create toroidal topology;

open boundaries may permit flux to enter or leave;

and mixed conditions may represent preparation and measurement contexts.

The global topology of K matters because a locally flat assignment can still possess nontrivial holonomy around a noncontractible loop. This distinction separates local curvature from global topological sector.

A configuration may satisfy:

H_f = id

for every elementary face while retaining nontrivial transport around a loop that cannot be contracted through the faces of K.

The model must therefore distinguish:

local face curvature;

global holonomy;

boundary-induced obstruction;

and localized defect structure.

The finite 2-complex provides a controlled environment in which all of these can be calculated exactly.

The first computational studies should begin with simple complexes whose structure is fully transparent:

a single polygonal face;

two faces sharing an edge;

a rectangular grid;

a triangulated disk;

a cylinder;

a torus;

and a sphere with a puncture or prescribed defect.

These cases allow local coherence, topology, boundary conditions, and gauge redundancy to be separated systematically.

The role of K can be summarized as follows:

vertices support local identity;

edges support transport;

faces test closure;

cycles support topology;

boundaries encode physical constraints.

The complex is therefore not merely a numerical lattice. It is the compositional skeleton on which equivalence becomes a field.

2.2 The Groupoid of Local Identity

The second foundational ingredient is a groupoid G of local identity structures.

A groupoid consists of:

a collection of objects;

a collection of morphisms between objects;

identity morphisms;

composition of compatible morphisms;

and inverses for every morphism.

For objects A, B, and C, a morphism:

u : A -> B

may be composed with:

v : B -> C

to form:

v o u : A -> C.

Every object A possesses an identity morphism:

id_A : A -> A.

Every morphism u has an inverse:

u^-1 : B -> A,

satisfying:

u^-1 o u = id_A,

u o u^-1 = id_B.

The groupoid is interpreted as the finite reversible architecture of physical identification.

Its objects represent admissible local identity types. A local identity type may be understood as the structure that can persist, transform, and be recognized within one local context.

The word object must be interpreted carefully. It does not necessarily mean a particle, field value, or material constituent. It is a mathematical representative of the local organization whose transformations are being modeled.

Depending on the application, an object may encode:

a local internal state space;

a representation type;

a phase of matter;

a local algebra of observables;

a bundle fiber type;

a boundary condition;

a symmetry-breaking sector;

or a coarse-grained identity pattern.

A morphism:

u : A -> B

represents an admissible reversible identification of A with B.

It states that the identity structure presented locally as A can be transported or reexpressed as B without loss of the physically relevant organization.

The existence of such a morphism does not mean that A and B are symbolically identical. It means that they are structurally equivalent according to the admissibility criteria of the model.

This distinction permits multiple local presentations of one physical identity.

There may also be several distinct morphisms from A to B:

u, v : A -> B.

These morphisms represent different ways of identifying the same two local identity types.

They may differ by:

phase;

orientation;

winding;

path history;

internal symmetry;

or another transformation carrying physical or topological information.

An ordinary equivalence relation would record only that A is equivalent to B. The groupoid records the full space of explicit equivalences.

For each object A, the morphisms from A to itself form the automorphism group:

Aut(A) = Hom_G(A,A).

This group contains the reversible self-transformations of the local identity structure.

The automorphism group plays a central physical role. In the proposed interpretation, it is the local source of gauge symmetry.

A change of presentation at a vertex is represented by:

g in Aut(A).

The object remains of type A, but its local representative changes under one of its self-equivalences.

Thus:

gauge symmetry = local automorphism of identity.

This relation is not merely terminological. It provides a structural explanation of why gauge transformations form groups. They are the reversible self-maps of a local identity object.

The groupoid framework is more general than the use of a single group. A group can be viewed as a groupoid with one object. If G has one object A, then every morphism is an automorphism of A:

Hom_G(A,A) = Aut(A) = G.

This recovers the standard setting of gauge theory in which a fixed group acts at every site.

A many-object groupoid permits a richer theory.

Suppose G contains objects:

A_1, A_2, ..., A_m.

Each object may possess a different automorphism group:

Aut(A_1), Aut(A_2), ..., Aut(A_m).

Morphisms may connect some pairs of objects and not others.

This creates several possible structures:

objects connected by morphisms belong to the same groupoid component;

objects in disconnected components represent identity types that cannot be reversibly identified;

automorphism groups characterize local self-symmetry;

and morphism multiplicities encode different modes of equivalence.

A connected groupoid can contain several different object presentations while representing one broad equivalence sector. All automorphism groups of objects in the same connected component are isomorphic, although not necessarily canonically identical.

If:

u : A -> B,

then u induces an isomorphism:

Aut(A) -> Aut(B)

given by:

g -> u o g o u^-1.

This shows that gauge symmetry can be transported between equivalent local identity presentations.

The isomorphism depends on the chosen equivalence u. Different choices may differ by conjugation. This dependence is physically appropriate because the identification of local symmetry frames is itself part of the transport structure.

Disconnected components of G represent distinct identity sectors. No reversible morphism connects objects belonging to different components.

Transitions between such sectors cannot be represented as gauge equivalences within the groupoid. They would require either:

noninvertible morphisms in a larger category;

an enlargement of the groupoid;

a defect boundary;

a phase transition;

or a change in the admissibility structure.

This distinction separates equivalence from physical transformation.

Within one connected component, changes may represent different presentations of the same broad identity type. Between disconnected components, the model records a genuine distinction unless a larger process category supplies a transition.

The present paper focuses first on reversible transport. Consequently, the groupoid G captures the gauge-equivalence structure, while irreversible or identity-changing processes are postponed to a broader categorical extension.

The groupoid may be finite or compact.

A finite groupoid is especially suitable for exact computation. One can enumerate:

all objects;

all morphisms;

all configurations;

all gauge transformations;

all gauge orbits;

all stabilizers;

and the complete partition function.

A compact Lie groupoid permits closer contact with conventional continuum gauge theory but introduces analytic complications.

The initial constructive model should therefore begin with finite groupoids.

Examples include:

a one-object groupoid based on a finite cyclic group;

a one-object groupoid based on a finite non-Abelian group;

an action groupoid generated by a group acting on a finite set;

a connected groupoid with several equivalent object presentations;

a disjoint union of components with different automorphism groups;

or a groupoid generated by a finite system of relations.

These examples can test progressively more ambitious claims.

A cyclic group can test Abelian holonomy and flux.

A non-Abelian group can test conjugation, path ordering, and noncommuting curvature.

A many-object connected groupoid can test presentation changes between different local identity representatives.

A heterogeneous disconnected groupoid can test competition among distinct identity sectors.

The groupoid is not assumed to contain every mathematically possible equivalence. It contains those transformations admitted by the model.

This restriction is essential. If every formal isomorphism were treated as physical, the theory could collapse physically meaningful distinctions.

The physical groupoid should therefore ultimately be written as:

G_phys,

to emphasize that its morphisms are admissible physical identifications.

The criteria of admissibility will be developed later. At this stage, the groupoid supplies the reversible structural possibilities from which field configurations are built.

The groupoid interpretation can be summarized:

objects = local identity presentations;

morphisms = admissible reversible identifications;

automorphisms = local gauge symmetries;

connected components = reversible identity sectors;

loops = self-equivalences;

composition = sequential transport;

inverses = reversibility.

The finite oriented 2-complex tells us where comparisons occur. The groupoid tells us what can be compared and how.

Together, they define the arena of the theory.

2.3 Field Configurations

A field configuration assigns groupoid data to the cells of the finite complex.

Let K be the finite oriented 2-complex and G the groupoid of local identity structures.

A configuration Phi consists initially of:

an object A_v in Ob(G) for each vertex v in V;

an invertible morphism U_e in Mor(G) for each oriented edge e in E;

subject to the condition:

U_e : A_s(e) -> A_t(e).

Thus the object assignments determine which local identity type is present at each vertex, while the edge assignments determine how neighboring identity types are identified.

For the reversed edge:

e^-1,

the transport is required to satisfy:

U_(e^-1) = U_e^-1.

The complete configuration may be written:

Phi = ({A_v},{U_e}).

This is the minimal equivalence field.

Unlike a conventional lattice gauge field, the edge morphism U_e need not always belong to one fixed group. Its source and target may be different objects.

In the one-object case:

A_v = A

for every vertex, and:

U_e in Aut(A).

This is the ordinary lattice gauge situation.

In the many-object case, the field includes both:

the local identity assignment A_v;

and the transport assignment U_e.

This allows identity type itself to vary across the complex.

However, a morphism U_e can exist only if A_s(e) and A_t(e) lie in the same connected component of the groupoid. Therefore, adjacent vertices connected by admissible reversible transport must carry equivalent identity objects.

If neighboring vertices carry objects in different disconnected components, then one of several interpretations is required:

the edge is forbidden;

the configuration has infinite action;

the edge represents a noninvertible process outside the groupoid;

or a domain wall or phase boundary is present.

The minimal reversible model may simply restrict allowed configurations to those for which every edge morphism exists.

A more general model can include identity-changing defects at forbidden or singular interfaces.

A configuration can also include local matter data.

For each vertex v, let:

psi_v in V_(A_v),

where V_(A_v) is a vector space, module, Hilbert space, or other representation object associated with the local identity type A_v.

For each groupoid morphism:

u : A -> B,

there is a corresponding representation map:

rho(u) : V_A -> V_B.

Compatibility with groupoid composition requires:

rho(v o u) = rho(v) o rho(u),

rho(id_A) = id_(V_A),

rho(u^-1) = rho(u)^-1.

The assignment rho is therefore a functor from the groupoid G into a category of representation spaces.

This functor gives operational meaning to the transport morphisms. A local matter variable psi at the source vertex is transported to the target representation space by:

rho(U_e) psi_s(e).

The covariant difference along an edge is then:

D_e psi = psi_t(e) - rho(U_e) psi_s(e).

This quantity compares the actual local field value at the target with the value predicted by coherent transport from the source.

When:

D_e psi = 0,

the matter data are covariantly constant along the edge.

When:

D_e psi != 0,

the local matter field fails to agree with transported identity.

This failure contributes to the matter action.

The use of representation spaces is optional in the first pure-transport model. The gauge sector can be studied without matter variables. However, matter representations are necessary for defining charges, interactions, and excitation spectra.

The full configuration may therefore be written:

Phi = ({A_v},{U_e},{psi_v}).

Additional data may later include:

2-morphisms on faces;

local scalar order parameters;

fermionic variables;

edge lengths;

causal labels;

face amplitudes;

or higher-cell coherence witnesses.

The present model should proceed in layers.

The first layer contains fixed identity objects and variable edge transports.

The second layer allows several equivalent object presentations.

The third layer allows competing identity sectors.

The fourth layer adds matter representations.

The fifth layer adds higher morphisms and higher gauge structure.

This staged construction prevents the model from becoming formally overloaded before its simplest behavior is understood.

A configuration is not itself a physical state. It is a presentation of a physical configuration.

Different assignments may represent the same physical situation because local automorphisms can change the object representatives and edge transports without changing the invariant content.

The physically meaningful object is therefore the gauge orbit or, more accurately, the configuration groupoid whose:

objects are field assignments;

morphisms are local changes of presentation.

This distinction is fundamental.

The raw configuration set contains descriptive redundancy.

The physical moduli structure retains:

which configurations are equivalent;

which transformations connect them;

which configurations have stabilizers;

and which sectors remain disconnected.

The action must be defined on raw configurations but descend to the physical moduli structure.

A useful formal description views a configuration as a functor from the path groupoid of the complex into G.

Let:

Pi_1(K^(1))

denote the path groupoid of the 1-skeleton of K. Its objects are vertices, and its morphisms are edge paths modulo immediate cancellation of an edge with its inverse.

A transport assignment defines a functor:

Phi_1 : Pi_1(K^(1)) -> G.

It sends:

each vertex v to A_v;

each edge e to U_e;

each composed path to the corresponding composite morphism;

each reversed path to the inverse morphism.

This formulation automatically enforces compositional consistency along paths.

The faces then impose additional information. If every face boundary is required to map to the identity, the functor factors through the fundamental groupoid of the full 2-complex.

In the present theory, however, nontrivial face holonomy is allowed and interpreted as curvature. The face boundaries therefore measure the failure of the edge functor to descend to a flat representation of the complete complex.

This makes curvature a failure of factorization.

A flat configuration assigns identity holonomy to every contractible face boundary.

A curved configuration assigns nontrivial automorphisms to some face boundaries.

A topologically nontrivial flat configuration may have trivial elementary face holonomies but nontrivial holonomy around global noncontractible cycles.

The field configuration thus contains three distinguishable forms of structure:

local object identity;

edge transport;

loop obstruction.

The first specifies what is being transported.

The second specifies how it is transported.

The third records whether transport composes coherently around loops.

This triad is the finite core of Univalent Relational Gauge Dynamics.

2.4 Why a Groupoid Rather Than a Single Group

The most immediate question is why the model requires a groupoid rather than an ordinary gauge group.

A single group is sufficient for conventional lattice gauge theory. One chooses a gauge group G and assigns an element:

U_e in G

to every oriented edge.

A gauge transformation assigns:

g_v in G

to every vertex and acts by:

U_e -> g_t(e) U_e g_s(e)^-1.

This construction is mathematically powerful and empirically successful.

The purpose of the groupoid extension is not to replace it unnecessarily. It is to expose assumptions that conventional gauge theory takes as fixed.

A single gauge group assumes that:

the same internal identity type exists at every vertex;

the same automorphism group acts everywhere;

the available symmetry structure is fixed in advance;

and local variations are changes of frame within one globally selected identity sector.

These assumptions are reasonable when the gauge theory is already known. They are less satisfactory in a foundational theory seeking to explain why a particular gauge structure exists.

A groupoid permits the local identity object itself to be represented explicitly.

The ordinary group case can be recovered when every vertex carries the same object A and all transports are automorphisms:

U_e in Aut(A).

The groupoid theory therefore contains ordinary gauge theory as the special case in which the object layer has been frozen.

This gives the conceptual correspondence:

group = self-equivalences of one fixed identity type;

groupoid = equivalences among possibly different local presentations of identity.

The groupoid adds several capabilities.

First, it distinguishes object identity from automorphism symmetry.

In ordinary gauge theory, the group is postulated. In the groupoid approach, the group appears as:

Aut(A).

This does not yet derive A or Aut(A), but it changes the explanatory target. The theory may seek a dynamics selecting identity objects whose automorphism structures reproduce observed gauge symmetries.

Second, a groupoid can represent several presentations of the same physical identity.

Suppose A and B are different objects connected by an equivalence:

u : A -> B.

Their automorphism groups are structurally equivalent through conjugation by u.

This allows the theory to distinguish:

literal sameness of presentation;

equivalence of identity type;

and transformation between equivalent local descriptions.

Such distinctions are central to an equivalence-first ontology.

Third, a groupoid preserves stabilizer information.

A quotient set of objects under equivalence records only the resulting classes. The groupoid also records the self-equivalences of each object.

These automorphisms affect:

symmetry;

state counting;

charge representations;

quantization weights;

and local degeneracy.

Two object sectors with the same number of equivalence classes may still possess different automorphism structures and therefore different physical behavior.

Fourth, a groupoid can describe heterogeneous local symmetry.

Different object types may have different automorphism groups. This opens the possibility of:

phase-dependent gauge structure;

symmetry enhancement;

symmetry reduction;

domain formation;

or dynamical sector selection.

For example, one object type may possess a large automorphism group while another possesses only a subgroup.

A transition in the dominant identity type could then appear as symmetry breaking.

Fifth, a groupoid can encode relations among distinct local fibers more naturally than a fixed group.

In geometry, gauge theory is often described using bundles whose fibers are locally identified but not globally identical in a preferred way. A groupoid treats the various local representatives and their transition morphisms as primary data.

This is closer to the actual local-to-global structure of gauge fields.

Sixth, the groupoid framework supports a direct connection to homotopy quotients and moduli stacks.

The physical configuration space of a gauge theory is not merely the set of gauge orbits. Configurations with nontrivial stabilizers possess additional symmetry data.

A groupoid representation retains this information automatically.

Seventh, a groupoid makes the possibility of emergent gauge symmetry mathematically expressible.

Suppose several identity sectors are allowed, with objects A_i and automorphism groups Aut(A_i). The partition function may assign different statistical weights to configurations dominated by different sectors.

If one sector minimizes free energy or maximizes the appropriate coherent measure, its automorphism group becomes the effective low-energy gauge symmetry.

This would not yet explain the precise Standard Model group, but it would provide a mechanism by which gauge symmetry could be selected rather than assumed.

Eighth, a groupoid provides the natural base for higher extension.

Higher gauge theory replaces groups by 2-groups and ordinary groupoids by higher groupoids. Beginning with a groupoid therefore creates a direct path toward:

gauge transformations between different presentations;

2-morphisms between transports;

surface holonomy;

higher-form gauge fields;

and coherence among alternative compositions.

The groupoid should nevertheless be used conservatively.

If every possible identity type and every possible equivalence are admitted, the theory becomes unconstrained. Its predictive value would disappear.

The physical content depends on:

which objects are included;

which morphisms are admissible;

which costs are assigned;

which boundary conditions are imposed;

and which sectors remain dynamically stable.

The groupoid is therefore not itself the physical law. It is the domain on which the physical law acts.

A useful comparison is:

the manifold does not determine the metric;

the Hilbert space does not determine the Hamiltonian;

the gauge group does not determine the action;

and the groupoid does not determine the coherence dynamics.

The action and admissibility selector provide the physical selection.

The justification for the groupoid is thus not that it is more abstract than a group. It is that it distinguishes structures that a foundational theory may need to explain.

A single group answers:

How does one fixed identity type transform under its self-symmetries?

A groupoid asks:

Which local identity types exist?

Which are physically equivalent?

How are their symmetry groups related?

Which sectors can coexist?

Which transitions are reversible?

Which identity structure is dynamically selected?

The first question defines conventional gauge theory. The second set of questions belongs to a theory seeking the origin of gauge structure itself.

2.5 Higher Extension

The finite groupoid model captures ordinary reversible equivalence, but the full equivalence-first program requires relations among transformations themselves.

Suppose two morphisms connect the same objects:

u, v : A -> B.

The groupoid treats u and v as distinct arrows. It does not yet specify whether they should themselves be regarded as equivalent transformations.

A higher theory introduces a 2-morphism:

alpha : u => v.

This 2-morphism represents an admissible deformation, identification, or coherence relation between two transports.

The presence of alpha states that u and v may be different presentations of the same higher process.

If no such 2-morphism exists, the two transports may belong to genuinely distinct process sectors even though they share the same source and target.

The hierarchy can continue.

Given:

alpha, beta : u => v,

a 3-morphism may relate alpha and beta.

Higher morphisms then relate those relations, continuing as required.

This structure leads from groupoids to 2-groupoids and ultimately to infinity-groupoids.

The present paper does not begin with the full infinity-groupoid because the foundational claims should first be tested in the simplest nontrivial model. A finite 1-groupoid already supports:

local automorphisms;

gauge transformations;

edge transport;

holonomy;

curvature;

topological sectors;

defect classes;

and exact groupoid-weighted quantization.

These features are sufficient for the first recovery theorem.

The higher extension becomes necessary when the physical equivalence of transports cannot be represented adequately by equality of groupoid morphisms.

Several situations motivate this extension.

First, alternative paths may implement transports that differ as morphisms but are related by a continuous deformation.

A 2-morphism can represent the deformation between those path transports.

Second, higher gauge theories contain gauge transformations between gauge transformations.

Ordinary gauge transformations may themselves possess redundancy. This gauge-of-gauge structure requires 2-morphisms.

Third, extended objects couple naturally to higher transport.

Point particles trace paths and couple to ordinary connections.

Strings sweep out surfaces and can couple to 2-form gauge fields.

Membranes and higher branes require higher-dimensional transport.

A 2-groupoid or higher groupoid provides the algebraic structure appropriate to such extended processes.

Fourth, weak composition requires explicit coherence.

In an ordinary category, composition is strictly associative:

h o (g o f) = (h o g) o f.

In a weak 2-category, these composites may not be literally equal. Instead, they are connected by an associator 2-morphism.

Different compositions of associators must satisfy higher coherence laws.

If physical processes compose only up to coherent equivalence, the higher morphisms are not optional bookkeeping. They are the structure that makes multi-stage physical identification consistent.

Fifth, local-to-global gluing may require higher compatibility.

On pairwise overlaps, local presentations may be connected by transition morphisms:

u_ij : A_i -> A_j.

On triple overlaps, one may not require strict equality:

u_jk o u_ij = u_ik.

Instead, there may be a specified 2-morphism:

alpha_ijk : u_jk o u_ij => u_ik.

On quadruple overlaps, the alpha_ijk must satisfy an additional coherence condition.

This hierarchy is characteristic of higher bundles, gerbes, and higher gauge fields.

Sixth, the physical interpretation of superposition and measurement may require higher paths.

If alternative identifications are connected by higher morphisms, interference may depend not only on the existence of paths but on the structure of transformations among them.

Measurement as truncation then becomes a map that deliberately forgets some higher morphism levels.

The finite groupoid model can approximate this structure by counting alternative paths and holonomies, but a genuine HoTT interpretation ultimately requires higher identity types.

A finite 2-groupoid extension can be built by assigning:

objects A_v to vertices;

1-morphisms U_e to edges;

2-morphisms B_f to faces.

The face 2-morphism may compare the composite boundary transport with a chosen reference morphism.

For a face f based at v, one may write schematically:

B_f : H_f => id_(A_v),

when a coherent trivialization exists.

If no such B_f exists, the face carries a higher obstruction.

Alternatively, B_f may be an independent surface variable whose boundary is related to the edge holonomy.

The precise formulation depends on whether the model uses:

strict 2-groups;

crossed modules;

weak 2-groupoids;

bicategories;

or another finite higher-categorical structure.

A crossed module provides one practical finite realization.

Let G and H be groups with a homomorphism:

partial : H -> G

and an action of G on H satisfying compatibility conditions.

The G-valued edge variables represent ordinary transport, while H-valued face variables represent higher transport.

The fake-curvature condition relates the boundary holonomy of the face to the image of its H-variable.

Such models provide a bridge between the present groupoid field theory and higher gauge theory.

However, the full higher structure should be introduced only after several questions are answered in the 1-groupoid model:

Can the action be defined unambiguously?

Does it descend to gauge equivalence classes?

Can the partition function be computed?

Do stable defects exist?

Does a nontrivial mass gap appear?

Can heterogeneous automorphism sectors be selected?

If the answers are negative at the first level, adding higher structure may merely conceal the failure.

If the answers are positive, higher morphisms can deepen the theory in a controlled way.

The relation to Homotopy Type Theory is then direct.

In HoTT:

objects correspond to terms of a type;

1-morphisms correspond to identity paths;

2-morphisms correspond to identities between paths;

higher morphisms correspond to higher identity types.

The groupoid model is the 1-truncated approximation of this hierarchy.

It retains points and invertible paths but discards nontrivial paths between paths.

A 2-groupoid model retains one additional level.

An infinity-groupoid retains the full hierarchy.

The finite-observation principle suggests that experimentally accessible physics may often depend only on a truncation of the complete higher structure. The groupoid model may

therefore be more than a convenient approximation. It may represent the level of equivalence retained in a particular physical regime.

The hierarchy can be written:

set = identity classes only;

groupoid = objects and equivalences;

2-groupoid = equivalences and equivalences between them;

infinity-groupoid = complete higher identity structure.

The present model begins at the groupoid level because that is the minimal level at which gauge symmetry and automorphism data survive quotienting.

The higher extension will be considered successful only if it adds calculable physical content, such as:

new topological sectors;

higher-form charges;

surface observables;

additional interference structure;

anomaly cancellation conditions;

or constraints on admissible gauge configurations.

The use of higher category theory must not become ornamental. Each higher level must correspond to a definite transformation, observable, obstruction, or computational consequence.

The mathematical arena is therefore intentionally layered.

At the foundational level lies a finite complex of local contexts.

On this complex lies a groupoid-valued transport field.

Its automorphisms generate gauge freedom.

Its path composites generate holonomy.

Its nontrivial loops generate curvature and defects.

Its representations support matter.

Its moduli groupoid supports quantization.

Its higher extension introduces transformations among transports and increasingly refined coherence.

This layered arena is sufficient to convert the general language of equivalence-first physics into a concrete field theory.

The next section defines the local gauge transformations and the physical equivalence relation acting on these configurations.

3. Physical Equivalence and Gauge Freedom

3.1 Local Changes of Presentation

The field configurations defined in the preceding section contain descriptive redundancy. The same physical arrangement may be represented by different local choices of identity frame at each vertex. A constructive theory must distinguish these changes of presentation from genuine changes in physical content.

Let a field configuration be:

Phi = ({A_v},{U_e},{psi_v}).

For each vertex v, choose an automorphism:

g_v in Aut(A_v).

The collection:

g = {g_v | v in V}

defines a local change of presentation.

The object A_v is not replaced by a physically different identity type. Rather, its local representative is transformed by one of its admissible self-equivalences.

The edge transport:

U_e : A_s(e) -> A_t(e)

must transform compatibly with the changes of presentation at its source and target. The transformed transport is:

U_e' = g_t(e) o U_e o g_s(e)^-1.

This transformation law expresses a simple requirement. To describe the same transport in new local frames, one must first undo the new source frame, apply the original transport, and then apply the new target frame.

The order of composition is essential. For a non-Abelian automorphism structure, the factors cannot be rearranged arbitrarily.

The reversed edge transforms consistently:

U_(e^-1)' = g_s(e) o U_e^-1 o g_t(e)^-1.

Using:

U_(e^-1) = U_e^-1,

one finds:

U_(e^-1)' = (U_e')^-1.

Thus the orientation-reversal condition is preserved.

If matter data are present, they also transform under the local automorphisms.

Let:

psi_v in V_(A_v),

and let rho be the representation functor associated with the groupoid. Then:

psi_v' = rho(g_v) psi_v.

The transported source value transforms as:

rho(U_e') psi_s(e)'
= rho(g_t(e) o U_e o g_s(e)^-1) rho(g_s(e)) psi_s(e).

Functoriality gives:

rho(U_e') psi_s(e)'
= rho(g_t(e)) rho(U_e) psi_s(e).

The target field transforms as:

psi_t(e)' = rho(g_t(e)) psi_t(e).

Therefore the covariant difference:

D_e psi = psi_t(e) - rho(U_e) psi_s(e)

transforms as:

D_e psi' = rho(g_t(e)) D_e psi.

If the norm on the representation space is invariant under rho(g), then:

||D_e psi'||^2 = ||D_e psi||^2.

This establishes gauge invariance of the standard matter-transport term.

The local changes of presentation form a gauge group associated with the entire configuration:

Gauge(Phi) = product over v in V of Aut(A_v),

provided the object assignments are fixed.

In a one-object model with:

A_v = A

for all v, this becomes:

Gauge(Phi) = Aut(A)^|V|.

In a many-object model, different vertices may carry different automorphism groups, so the gauge group is a vertex-dependent product.

The word gauge is used here in its structural sense. A gauge choice is a choice of local representative within the same physical identity structure. Gauge transformation does not create a new physical world. It changes the local language in which the same relational configuration is described.

This interpretation clarifies the role of the vertex objects.

The object A_v identifies the local type of structure.

The automorphism g_v changes its presentation.

The edge morphism U_e compares presentations at neighboring vertices.

The transformed edge morphism U_e' expresses the same comparison after both endpoints have been re-described.

Gauge redundancy therefore appears because local descriptions are not absolute. The physically meaningful content must be encoded in quantities invariant under all admissible families {g_v}.

A local gauge transformation may vary independently at different vertices. This locality is crucial.

If the same automorphism g were applied at every vertex, the transformation would be global. A global symmetry may relate physically distinct states or generate conserved quantities.

A local gauge transformation, by contrast, changes the descriptive frame separately at each vertex while preserving the relational content of the complete configuration.

The distinction between local and global transformation depends on boundary conditions.

If the values g_v are constrained to equal the identity on the boundary, then the transformation is a pure interior gauge redundancy.

If nontrivial transformations are permitted at the boundary, they may act on boundary data and generate physical charges or edge degrees of freedom.

Thus a transformation that is gauge in the interior may become physically observable at a boundary.

This distinction is well known in gauge theory and becomes especially important in the present framework because physical identity is defined relationally. Boundary contexts determine which equivalences are treated as redundant and which are treated as physically distinguishable.

The action of the gauge group on field configurations may possess stabilizers.

A gauge transformation g stabilizes Phi when:

g · Phi = Phi.

Explicitly, for every edge e:

g_t(e) o U_e = U_e o g_s(e),

and, when matter is present:

rho(g_v) psi_v = psi_v.

The stabilizer group is:

Aut(Phi) = {g | g · Phi = Phi}.

This group records the self-equivalences of the complete field configuration.

It plays several roles:

it identifies enhanced symmetry;

it affects orbit size;

it enters groupoid-weighted quantization;

it distinguishes generic from exceptional configurations;

and it may encode residual gauge symmetry after symmetry breaking.

A configuration with trivial stabilizer has no nonidentity local presentation change that leaves every assigned field unchanged.

A configuration with a nontrivial stabilizer possesses additional self-symmetry.

The presence of stabilizers is one reason that the physical configuration space should not be treated merely as a set of gauge orbits. Two orbits may each correspond to one set-theoretic point while possessing different internal automorphism structures.

The local gauge action can be summarized as:

A_v remains the same identity type;

psi_v changes by the local representation of Aut(A_v);

U_e changes by source-target conjugation;

holonomy changes by conjugation;

and physical observables remain invariant.

This is the basic transformation law of Univalent Relational Gauge Dynamics.

3.2 Gauge Orbit and Physical Configuration

A raw field configuration is a mathematical presentation. The corresponding physical configuration is obtained only after local changes of presentation have been accounted for.

Let:

Conf(K,G)

denote the set or space of all allowed field configurations on the complex K with values in the groupoid G.

The gauge transformations act on this space.

For a configuration Phi and a gauge transformation g, write:

Phi' = g · Phi.

Two configurations are gauge equivalent when there exists an admissible g such that:

Phi' = g · Phi.

This defines an equivalence relation on raw configurations.

The simplest physical state space would be the quotient set:

M_set = Conf(K,G) / Gauge.

Each element of M_set is a gauge orbit.

However, the quotient set forgets important information.

It remembers that Phi and Phi' are equivalent, but it forgets:

which transformation relates them;

how many transformations relate them;

whether the transformations form disconnected families;

whether Phi has a nontrivial stabilizer;

and how gauge transformations compose.

For this reason, the more appropriate physical structure is the action groupoid:

M = Conf(K,G) // Gauge.

Its objects are raw field configurations.

Its morphisms are gauge transformations between configurations.

A morphism:

g : Phi -> Phi'

exists when:

Phi' = g · Phi.

Composition is composition of local gauge transformations.

Every morphism is invertible because each g_v is an automorphism.

The action groupoid therefore retains the complete reversible equivalence structure of the field theory.

The notation:

Conf(K,G) // Gauge

denotes a homotopy quotient rather than an ordinary quotient.

The distinction is physically significant.

Suppose two configurations have the same gauge-invariant action value but different stabilizer groups.

The quotient set may treat each as one point.

The action groupoid distinguishes them because their automorphism structures differ.

This difference affects the weighting of configurations in a finite partition function.

A configuration with a large stabilizer has fewer distinct presentations in its orbit than a generic configuration with the same ambient gauge group.

If every raw presentation were counted equally without correction, highly symmetric configurations would be weighted incorrectly.

The groupoid description resolves this through automorphism-aware counting.

For a finite gauge group, the size of an orbit satisfies:

|Orbit(Phi)| = |Gauge| / |Aut(Phi)|.

This is the orbit-stabilizer relation.

Summing over all raw configurations and dividing by the total gauge-group size is equivalent, under suitable finite conditions, to summing over gauge orbits weighted by inverse stabilizer order.

Thus:

Z = (1 / |Gauge|) sum over Phi exp[-S(Phi)/hbar]

can be rewritten as:

Z = sum over [Phi] exp[-S([Phi])/hbar] / |Aut(Phi)|,

up to conventions associated with the finite configuration groupoid.

This formula displays explicitly why the physical configuration space is not merely a set.

Its points carry internal symmetry.

The gauge orbit should not be confused with a dynamical trajectory.

Configurations within one orbit are different presentations of the same physical situation.

A trajectory or physical transition connects configurations that are not necessarily gauge equivalent.

This distinction can be expressed as:

gauge morphism = reversible change of presentation;

dynamical morphism = physical process or evolution.

The action groupoid contains the first kind.

A complete theory of time and irreversible process may require a larger category containing both gauge equivalences and noninvertible physical transitions.

Within the present paper, the physical configuration at one stage is represented by an object of the moduli groupoid M.

The gauge orbit of Phi is:

[Phi] = {g · Phi | g in Gauge}.

But the physical identity of [Phi] includes more than membership in this set. It includes:

the stabilizer Aut(Phi);

the arrangement of gauge transformations;

the topology of the orbit;

and, in a higher extension, higher relations among gauge transformations.

This enriched view is essential for the equivalence-first interpretation.

Physical identity is not produced by erasing all descriptive distinctions. It is represented by the structured network that explains why those distinctions do not alter physical content.

A useful distinction should also be made between small and large gauge transformations.

In a continuous theory, small gauge transformations are connected to the identity, while large gauge transformations belong to disconnected components of the gauge-transformation space.

In a finite model, connectedness may be replaced by algebraic or topological sector structure.

Some gauge transformations may act trivially on all observables and represent pure redundancy.

Others may alter global winding, boundary data, or topological charge and therefore relate physically distinguishable sectors.

The classification depends on the admissibility and boundary rules of the model.

The physical quotient must therefore be defined with care.

If transformations that change measurable boundary data are incorrectly divided out, genuine physical distinctions will be erased.

If pure changes of local frame are not divided out, descriptive redundancy will be mistaken for physical multiplicity.

The moduli groupoid must encode exactly the correct equivalence notion.

This leads directly to the problem of admissibility.

3.3 Admissibility

The existence of a mathematical isomorphism does not automatically imply physical equivalence.

A physical theory must specify which transformations preserve the structures that define the physical situation.

Let:

Equiv_math(A,B)

denote the full set or space of mathematical equivalences between A and B.

Define:

Equiv_phys(A,B) subset or equal to Equiv_math(A,B)

as the admissible physical equivalences.

The admissibility selector determines which transformations count as changes of presentation and which count as genuine physical changes.

This is one of the most important components of the theory.

Without a restrictive admissibility condition, the equivalence-first program could identify structures that are mathematically related but physically distinct.

For example, two field configurations might be isomorphic as bare algebraic objects while differing in:

energy;

causal structure;

boundary conditions;

orientation;

topological charge;

interaction rules;

or observable probabilities.

Such configurations should not be identified merely because an abstract isomorphism exists after relevant structure has been forgotten.

Physical equivalence must be structure-preserving.

The preserved structure may include:

source and target compatibility;

composition;

orientation;

representation content;

boundary data;

observable values;

causal relations;

locality;

topological sector;

action value;

or experimental predictions.

The exact criteria depend on the level of theory.

At minimum, admissible equivalences should satisfy several closure properties.

Identity:

id_A belongs to Equiv_phys(A,A).

Inversion:

if u belongs to Equiv_phys(A,B), then u^-1 belongs to Equiv_phys(B,A).

Composition:

if u belongs to Equiv_phys(A,B) and v belongs to Equiv_phys(B,C), then v o u belongs to Equiv_phys(A,C).

These conditions ensure that admissible equivalences form a subgroupoid:

G_phys subset or equal to G_math.

The admissibility selector may be defined by preserved observables.

Suppose {O_i} is a family of physically meaningful quantities. Then u : A -> B may be admissible when:

O_i(A) is equivalent to O_i(B)

for every required observable, with the equivalence transported through u.

However, defining admissibility entirely through known observables risks circularity. The theory may seek to derive observables from the equivalence structure itself.

A deeper criterion may therefore require preservation of the physical action and compositional architecture.

One possible condition is:

S(u · Phi) = S(Phi)

for every configuration on which u acts as a change of presentation.

This identifies symmetries of the action.

But action invariance alone may also be too weak. Distinct physical states can possess equal action.

Admissibility must preserve not merely a numerical value but the relevant relational structure.

A practical finite model may define admissibility in stages.

First, specify a mathematical groupoid G of candidate equivalences.

Second, impose local constraints selecting a subgroupoid G_phys.

Third, define the action only on G_phys-valued configurations.

Fourth, test whether the resulting observables descend consistently.

The admissibility selector may depend on context.

A transformation may be admissible in the interior but not at the boundary.

A transformation may be admissible at low energy but distinguishable at higher resolution.

A transformation may preserve a coarse observable while changing microscopic structure.

Thus physical equivalence may be:

exact;

effective;

scale-dependent;

context-dependent;

or approximate.

The present paper begins with exact finite equivalence.

Approximate equivalence should be introduced only after the exact structure is understood.

An exact admissible equivalence preserves the full set of structures included in the model.

An approximate equivalence may preserve them only within a tolerance or over a restricted observational regime.

Scale-dependent equivalence could later be represented by a filtration:

G_phys^(0) subset or equal to G_phys^(1) subset or equal to ...

where different observational levels identify different amounts of structure.

This would connect naturally with coarse-graining and renormalization.

At coarse resolution, many microscopic presentations may become physically indistinguishable.

At finer resolution, distinctions may reappear.

The admissibility selector would then vary with scale.

For the initial finite gauge model, the admissibility rules should be explicit and conservative.

A morphism u : A -> B is admissible when it:

belongs to the chosen physical groupoid;

preserves the representation functor;

respects boundary conditions;

preserves orientation when orientation is physical;

and maps allowed configurations to allowed configurations.

If matter spaces are assigned, admissibility requires that rho(u) be a permitted isomorphism between the corresponding representation spaces.

If local potentials or interaction terms are assigned, admissibility requires compatibility with those structures.

If the model includes labels that are intended to represent physical charges, an admissible equivalence must preserve their representation class or transform them in the prescribed way.

The admissibility selector also determines whether two object types belong to the same physical sector.

If:

Equiv_phys(A,B) is empty,

then A and B are physically distinct identity types under the current theory.

If it is nonempty, they are different presentations of one reversible identity sector.

If several inequivalent admissible morphisms connect them, the space of physical identifications has nontrivial internal structure.

In a higher theory, Equiv_phys(A,B) may itself be a groupoid or higher type.

The finite groupoid model retains only the first level.

The theory must also distinguish admissibility from dynamical accessibility.

Two configurations may be physically distinct yet dynamically connected through a noninvertible process.

For example, a decay may transform one particle type into others.

A phase transition may connect states belonging to different reversible identity sectors.

Such processes should not be placed inside the gauge groupoid merely because they are physically possible.

Gauge equivalence answers:

Do these presentations represent the same physical identity?

Dynamics answers:

Can one physical identity change into another?

Conflating these questions would erase the distinction between sameness and change.

The admissibility selector therefore defines the reversible identity structure. A larger process category may later contain noninvertible morphisms connecting distinct sectors.

The scientific importance of admissibility cannot be overstated.

If the selector is chosen arbitrarily to force desired results, the theory becomes unfalsifiable.

The paper must therefore state:

which admissibility rules are axiomatic;

which are derived from consistency;

which are selected by the action;

and which are tested computationally.

A successful theory would eventually derive much of the admissibility structure from more primitive principles.

At the present stage, admissibility is an explicit input whose consequences can be studied.

The central requirement is:

equivalence must preserve exactly the structures whose variation would constitute physical change.

3.4 Observable Descent

A quantity defined on raw configurations is physically meaningful only if it does not depend on arbitrary local presentation choices.

Let:

O : Conf(K,G) -> X

be a candidate observable, where X is a set, vector space, category, probability space, or another target structure.

The simplest descent condition is:

O(g · Phi) = O(Phi)

for every admissible gauge transformation g.

Such an observable is gauge invariant.

It factors through the quotient:

Conf(K,G) -> Conf(K,G) / Gauge.

Equivalently, there exists a function:

O_phys : M_set -> X

such that:

O = O_phys o q,

where q maps a raw configuration to its gauge orbit.

In a higher or categorical setting, strict equality may be too strong. The target X may itself possess equivalences.

Then the descent condition becomes:

O(g · Phi) is equivalent to O(Phi),

with coherence under composition of gauge transformations.

The observable must define a functor from the configuration groupoid to the target category.

This means:

objects Phi are assigned observable values O(Phi);

gauge morphisms g : Phi -> Phi' are assigned equivalences O(g);

identity gauge transformations map to identity equivalences;

and composition is preserved.

A set-valued gauge-invariant observable is the simplest special case, because every gauge morphism acts trivially on the resulting value.

Several natural observables arise in the finite model.

The conjugacy class of face holonomy is gauge invariant.

The trace of a represented holonomy is gauge invariant.

The product of transport around a closed loop gives a Wilson-loop observable.

The spectrum of rho(H_f) is gauge invariant.

The defect class associated with global holonomy may be gauge invariant.

The total action is gauge invariant when constructed from invariant local terms.

The partition function is gauge invariant when equivalence classes are counted correctly.

By contrast, an individual edge variable U_e is generally not gauge invariant.

Its value depends on the local frames chosen at its endpoints.

A local matter field psi_v is also not generally gauge invariant.

Only appropriate combinations, norms, contractions, or relational comparisons descend to physical observables.

This distinction does not make gauge-dependent quantities useless.

They remain essential computational variables.

Coordinates, potentials, frames, and local representatives often simplify calculations.

Their physical interpretation arises only through combinations that descend.

The observable-descent principle may be expressed diagrammatically:

Conf(K,G) --O--> X
|
q
|
v
M_phys -------> X.

The lower map exists only when O is constant or coherently equivalent on gauge orbits.

The face-holonomy example illustrates descent directly.

For a face f based at vertex v:

H_f in Aut(A_v).

Under a gauge transformation:

H_f' = g_v o H_f o g_v^-1.

Therefore H_f itself is not invariant as a specific group element unless the group is Abelian or H_f lies in the center.

But its conjugacy class:

[H_f]_conj

is invariant.

Likewise, for a representation rho:

Tr rho(H_f')

= Tr[rho(g_v) rho(H_f) rho(g_v)^-1]

= Tr rho(H_f).

Thus:

W_f = Tr rho(H_f)

is a gauge-invariant observable.

More generally, for a closed loop gamma:

W_gamma = Tr rho(U_gamma)

defines a Wilson-loop observable.

These quantities probe the relational field without depending on the local choice of frame.

Open-path transport can also become observable when its endpoints carry transforming matter fields.

For a path p from v to w, consider a bilinear expression:

psi_w^dagger rho(U_p) psi_v.

Under gauge transformation, the endpoint transformations cancel when the inner product is invariant.

This yields a gauge-invariant relational observable connecting two local contexts.

The physical lesson is important:

an isolated local field value may be gauge dependent;

a relational comparison between appropriately transformed endpoints may be physical.

This is exactly what an equivalence-first theory should predict.

Observables are not primitive labels attached to isolated states. They are constructions that survive or coherently transform across the admissible equivalence structure.

Descent must also be checked globally.

A quantity may be locally gauge invariant yet fail to define a global observable because its local definitions do not agree on overlaps.

For example, locally defined phases, potentials, or sections may require transition functions to glue consistently.

The local-to-global problem is therefore part of observable descent.

In a finite complex, gluing can be checked combinatorially.

One assigns local observable data to subcomplexes and verifies compatibility on intersections.

In a higher theory, these compatibility conditions may hold only up to specified higher morphisms.

The requirement then becomes higher descent.

The present model begins with strict finite descent but is intended to extend naturally to:

sheaf-valued observables;

stack-valued fields;

higher holonomy;

and coherent local-to-global reconstruction.

The descent requirement also places constraints on proposed physical definitions.

If mass is defined as minimum defect coherence cost, it must be invariant under gauge transformation.

If charge is defined through an automorphism representation, the representation class must be independent of arbitrary local frame choice.

If distance is defined from minimum transport cost, equivalent path presentations must yield the same infimum.

If entropy is defined from equivalence-fiber volume, the counting must correct for automorphism redundancy.

If measurement is defined as truncation, the truncation map must be well defined on the physical moduli object rather than on arbitrary raw representatives.

Thus observable descent is not a secondary technical condition. It is the criterion separating physical quantities from descriptive artifacts.

3.5 First Theorem Target

The first rigorous result of the constructive theory should establish gauge invariance of holonomy-based observables and the coherence action.

The theorem is stated at the finite groupoid level.

Let K be a finite oriented 2-complex.

Let G be a finite or compact groupoid.

Let Phi assign an object A_v to each vertex and a morphism:

U_e : A_s(e) -> A_t(e)

to each oriented edge.

Let g = {g_v} be a local gauge transformation with:

g_v in Aut(A_v).

For a closed path:

gamma = e_n ... e_2 e_1

based at v, define the path holonomy:

H_gamma(Phi) = U_en o ... o U_e2 o U_e1.

Under the local gauge transformation:

U_e' = g_t(e) o U_e o g_s(e)^-1.

The theorem states:

H_gamma(g · Phi) = g_v o H_gamma(Phi) o g_v^-1.

The proof follows by substitution.

Write:

H_gamma(g · Phi)
= U_en' o ... o U_e2' o U_e1'.

Substituting the transformed edge variables gives:

H_gamma(g · Phi)
= (g_v o U_en o g_(v_(n-1))^-1)
o ...
o (g_(v_2) o U_e2 o g_(v_1)^-1)
o (g_(v_1) o U_e1 o g_v^-1).

Every intermediate factor cancels with its inverse.

The result is:

H_gamma(g · Phi)
= g_v o U_en o ... o U_e1 o g_v^-1.

Therefore:

H_gamma(g · Phi)
= g_v o H_gamma(Phi) o g_v^-1.

This establishes conjugation covariance of closed-path holonomy.

Now let:

K_A : Aut(A) -> R

be a conjugacy-invariant function for each object A, satisfying:

K_A(g h g^-1) = K_A(h).

Then:

K_(A_v)(H_gamma(g · Phi))
= K_(A_v)(H_gamma(Phi)).

Thus any conjugacy-invariant function of holonomy descends to the gauge quotient.

For each face f, let H_f be the face holonomy. Define:

S_face(Phi) = beta sum over f of w_f K_(A_vf)(H_f),

where v_f is the chosen base vertex of the face.

Then:

S_face(g · Phi) = S_face(Phi).

This is the first gauge-invariance theorem for the coherence action.

The choice of base vertex must not affect the physical face cost.

Changing the base vertex conjugates the face holonomy by the transport along the boundary segment connecting the two basepoints.

Therefore a conjugacy-invariant K also makes the face cost basepoint independent.

Orientation reversal must be handled separately.

For the oppositely oriented face:

H_(f^-1) = H_f^-1.

If the action is to be independent of arbitrary orientation convention, K should satisfy:

K(h^-1) = K(h).

Many standard class functions possess this property, including suitable trace-based expressions in unitary representations.

For example:

K(h) = Re Tr[I - rho(h)]

satisfies conjugacy invariance and inversion symmetry when rho is unitary.

The matter term can be included in the same theorem.

Let:

D_e psi = psi_t(e) - rho(U_e) psi_s(e).

Under the gauge transformation:

D_e psi' = rho(g_t(e)) D_e psi.

If the norm is invariant:

||rho(g)x|| = ||x||,

then:

||D_e psi'||^2 = ||D_e psi||^2.

Therefore:

S_matter = kappa sum over e of ||D_e psi||^2 + sum over v of V(psi_v)

is gauge invariant, provided the potential satisfies:

V(rho(g_v) psi_v) = V(psi_v).

A complete theorem may then state:

If the face costs are conjugacy invariant and inversion symmetric, the matter norms are representation invariant, the local potentials are automorphism invariant, and the boundary conditions are preserved by the admissible gauge transformations, then the total action S_URD is gauge invariant.

The theorem provides several immediate consequences.

First, the action descends from raw configurations to gauge-equivalence classes.

Second, the classical equations obtained by variation are compatible with gauge redundancy.

Third, the partition function may be defined over gauge orbits or the configuration groupoid.

Fourth, defect energies obtained by minimizing the action within a physical sector are presentation independent.

Fifth, computational gauge-invariance tests become possible.

The numerical implementation should generate random gauge transformations and verify that:

$S_{URD}(g \cdot Phi) - S_{URD}(Phi) = 0$

within exact arithmetic or numerical tolerance.

Likewise, the following quantities should remain invariant:

face-cost distributions;

Wilson loops;

defect classes;

partition-function contributions;

and proposed mass estimates.

The first theorem is intentionally modest.

It does not establish that the model describes nature.

It establishes that the central coherence action is well defined on physical equivalence classes rather than arbitrary presentations.

Without this result, the remainder of the theory would lack a stable foundation.

With it, the model can proceed to define transport, curvature, and defect structure in explicitly gauge-invariant terms.

The central conclusion of this section is:

physical configuration is not a raw assignment of objects and morphisms;

it is the complete gauge-equivalence structure of that assignment.

Local automorphisms encode changes of presentation.

Gauge orbits encode sameness of physical configuration.

Admissibility determines which equivalences are physical.

Observable descent determines which quantities survive changes of presentation.

The first theorem ensures that holonomy-based coherence costs meet this requirement.

The next section develops the geometry of path transport, face holonomy, and curvature as failed identity closure.

4. Identity Transport, Holonomy, and Curvature

4.1 Path Transport

The local data of the theory acquire physical meaning through composition along paths.

Let K be the finite oriented 2-complex introduced previously. A path p from a vertex v_0 to a vertex v_n is an ordered sequence of oriented edges:

p = (e_1, e_2, ..., e_n),

such that:

s(e_1) = v_0,

t(e_n) = v_n,

and:

t(e_i) = s(e_(i+1))

for every consecutive pair.

The edge assignments of a configuration provide morphisms:

U_ei : A_(v_(i-1)) -> A_(v_i).

The transport along the path is defined by composition:

U_p = U_en o ... o U_e2 o U_e1.

Thus:

U_p : A_(v_0) -> A_(v_n).

This composite is the identification obtained by carrying the local identity structure successively through the contexts encountered along the path.

The order of composition is essential. The first transport encountered acts first, so it appears on the right. In a non-Abelian or many-object groupoid, reversing the order generally changes the result.

Path transport satisfies several elementary properties.

For the trivial path at a vertex v:

id_v,

the transport is:

U_(id_v) = id_(A_v).

For two composable paths p from v to w and q from w to z, their concatenation q o p satisfies:

U_(q o p) = U_q o U_p.

For the reversed path:

p^-1 = (e_n^-1, ..., e_2^-1, e_1^-1),

the transport satisfies:

U_(p^-1) = U_p^-1.

These properties show that edge assignments define a functorial transport rule on the path groupoid of the 1-skeleton.

This functoriality is not yet the same as flatness.

Composition along a specified path is always defined. What may fail is path independence.

Suppose p and q are two different paths connecting the same vertices:

p,q : v -> w.

Their transports are:

U_p, U_q : A_v -> A_w.

If:

U_p = U_q,

then the two paths induce the same identification.

If:

U_p != U_q,

then the result of identity transport depends on the route.

This path dependence is the finite precursor of curvature, holonomy, or topological obstruction.

The comparison between the two path transports may be expressed by the loop:

q^-1 o p

based at v.

Its holonomy is:

U_(q^-1 o p) = U_q^-1 o U_p.

If this automorphism equals the identity, then the two transports agree.

If it is nontrivial, the two paths provide distinct identifications of the endpoint identity structure.

This observation gives a relational definition of curvature:

curvature measures the failure of alternative local identification routes to agree.

Path transport is transformed covariantly under local changes of presentation.

Let:

g_v in Aut(A_v)

be assigned at each vertex.

For a path p from v to w, the transformed path transport is:

U_p' = g_w o U_p o g_v^-1.

All transformations at intermediate vertices cancel.

Therefore an open-path transport is not gauge invariant by itself. Its source and target frames remain visible.

A closed-path transport based at v transforms by conjugation:

U_gamma' = g_v o U_gamma o g_v^-1.

This covariance is precisely what permits conjugacy classes, traces, spectra, and other class functions to define physical observables.

The interpretation of path transport extends beyond a formal gauge connection.

In conventional gauge theory, transport relates internal states at different spacetime points. In the present framework, the transported quantity is more general. It is the local structure by which an entity is identified across contexts.

The transport may compare:

different coordinate descriptions;

different gauge frames;

different local phases;

different representations of one object;

different experimental contexts;

or different stages of a persistent process.

Identity through change is therefore not assigned independently of transport. It is expressed by the existence and composition of the maps U_p.

If an entity is said to persist from v to w, the theory should be able to exhibit an admissible transport:

U_p : A_v -> A_w.

If no such transport exists, the two local presentations cannot be identified within the reversible structure of the model.

This formulation also separates local identity from global persistence.

An object A_v may possess a well-defined local structure, but its continuation across the complex depends on the transport assignments.

A global physical identity is not simply a repeated label placed at many vertices. It is a coherently transportable pattern.

The theory must therefore ask:

Does transport exist along every relevant path?

Do different paths agree?

Are disagreements removable by gauge transformation?

Do they define stable topological sectors?

Does transport preserve matter representations?

What cost is associated with maintaining it?

These questions are encoded in the holonomy structure.

For matter data, path transport acts through the representation functor.

Given:

psi_v in V_(A_v),

the transported field along p is:

rho(U_p) psi_v in V_(A_w).

The comparison with the actual endpoint field psi_w gives a path-level covariant discrepancy:

D_p psi = psi_w - rho(U_p) psi_v.

For a single edge, this reduces to the previously defined covariant difference.

For a longer path, D_p psi measures the cumulative failure of the field to be covariantly transported.

If D_p psi vanishes for all paths in a region and the transport is path independent, the matter configuration is covariantly constant in that region.

If it vanishes along individual paths but depends on the path chosen, the field may be locally transported while retaining nontrivial holonomy.

Path transport also provides the basis for relational distance.

If each transport morphism is assigned a cost C(U_e), then a path cost may be defined by:

C(p) = sum over e in p of C(U_e),

or by another composition-compatible rule.

A candidate distance between vertices is then:

d(v,w) = inf over p:v->w of C(p).

This construction will be developed later. At the present stage, it is sufficient to note that path transport supplies both identity comparison and the raw material from which effective geometry may emerge.

The central principle is:

a path is not merely a route through a preexisting space; it is a composable chain of physical identifications.

In a background-dependent theory, paths live inside geometry.

In the equivalence-first theory, effective geometry may arise from the network of paths and the costs of the transports assigned to them.

4.2 Face Holonomy

The first local test of path dependence occurs around a face.

Let f be an oriented face with boundary:

partial f = (e_1, e_2, ..., e_n).

Choose a base vertex:

v_f = s(e_1).

The face holonomy is the composite:

H_f = U_en o ... o U_e2 o U_e1.

Because the boundary is closed:

H_f : A_(v_f) -> A_(v_f).

Thus H_f belongs to the automorphism group:

H_f in Aut(A_(v_f)).

The face holonomy records the result of transporting the local identity structure around the elementary loop bounding the face.

If:

H_f = id_(A_(v_f)),

then transport closes trivially around that face.

If:

H_f != id_(A_(v_f)),

then the transported identity returns transformed.

The endpoint vertex is the same, but the identification path has accumulated a nontrivial automorphism.

This distinction is central to the theory.

Returning to the same presentation does not imply that nothing has occurred. A closed sequence of admissible transformations may produce:

a phase;

a rotation;

an internal symmetry action;

a winding;

a permutation;

or another self-equivalence.

The physical information resides in the loop, not merely in its endpoint.

The face holonomy depends on the chosen orientation.

For the reversed face:

f^-1,

the boundary is traversed in the opposite direction, and:

H_(f^-1) = H_f^-1.

A physically meaningful face cost should therefore either distinguish orientation for a stated reason or satisfy:

K(H_f^-1) = K(H_f).

The face holonomy also depends on the chosen base vertex as a specific automorphism.

Suppose another vertex w on the same boundary is chosen as the basepoint. Let P be the transport along the boundary from v_f to w. The holonomy based at w is:

H_f^(w) = P o H_f^(v_f) o P^-1,

with the exact convention determined by path orientation.

Thus changing the basepoint conjugates the holonomy.

The specific element H_f is presentation-dependent, but its conjugacy class is not.

Physical face observables should therefore be based on:

the conjugacy class of H_f;

the trace of a representation of H_f;

the spectrum of rho(H_f);

a central function of H_f;

or another conjugation-invariant quantity.

The simplest scalar face observable is:

$W_f = \mathrm{Tr}\ \rho(H_f)$.

In a unitary representation, one may instead use:

$K(H_f) = \mathrm{Re}\ \mathrm{Tr}[I - \rho(H_f)]$.

This quantity vanishes at the identity and is nonnegative under common normalizations.

The face holonomy has both local and global interpretations.

Locally, it measures failure of closure around one elementary face.

Globally, products of face holonomies may determine transport around larger contractible loops.

For an Abelian group, the holonomy of a large loop can often be expressed directly as the product of the elementary face holonomies inside it, with orientation signs.

For a non-Abelian group, path ordering and conjugation by connecting transports complicate the relation.

This noncommutativity is physically important. The order in which local curvature contributions are encountered affects the total transport.

A face with trivial holonomy is locally flat according to the transport field.

A collection of faces with trivial holonomy need not imply global triviality.

If the complex contains noncontractible loops, transport around them may remain nontrivial even though every elementary face is flat.

This distinction separates:

local curvature from global holonomy;

and geometric obstruction from topological sector.

For example, on a toroidal complex, one may assign commuting nontrivial holonomies around the two fundamental cycles while maintaining:

$H_f = \mathrm{id}$

for every contractible face.

Such a configuration is locally flat but globally nontrivial.

Its physical information cannot be detected by a single face cost. It requires observables around noncontractible cycles.

The theory should therefore include two classes of loop observables:

elementary face holonomies measuring local curvature;

global cycle holonomies measuring topological sector.

Boundary conditions can also force nontrivial face or cycle holonomy.

An imposed boundary transport may prevent the interior from being globally trivialized.

The resulting obstruction may localize into a defect or distribute across the complex, depending on the action.

The face holonomy is therefore not merely a diagnostic. It is the central local variable entering the coherence action.

The action assigns cost to the failure of identity transport to close.

This does not mean that every nontrivial holonomy is prohibited.

A nonzero holonomy may be:

energetically suppressed;

topologically required;

boundary induced;

quantized;

or stabilized by coupling to matter.

The action determines which nontrivial holonomies can persist.

The physical interpretation is:

identity transport is locally coherent when face holonomy is trivial or lies in a preferred low-cost class;

curvature is present when loop transport returns nontrivially;

defect structure arises when such nontriviality cannot be removed by admissible local transformation.

4.3 Transformation of Holonomy

The gauge transformation law for face holonomy follows from the transformation law of edge transport.

For each edge e:

U_e' = g_t(e) o U_e o g_s(e)^-1.

Let the face boundary be:

partial f = (e_1, ..., e_n),

based at v_f.

The transformed holonomy is:

H_f'
= U_en' o ... o U_e1'.

Substitution gives:

H_f'
= (g_(v_f) o U_en o g_(v_(n-1))^-1)
o ...
o (g_(v_2) o U_e2 o g_(v_1)^-1)
o (g_(v_1) o U_e1 o g_(v_f)^-1).

Every intermediate local transformation cancels with its inverse.

Therefore:

H_f' = g_(v_f) o H_f o g_(v_f)^-1.

This result has several consequences.

First, the statement:

H_f = id

is gauge invariant.

If H_f is the identity in one presentation, then:

H_f'
= g id g^-1
= id.

Thus local flatness does not depend on gauge choice.

Second, the conjugacy class of H_f is gauge invariant.

Gauge transformations may alter the representative but cannot change its class.

Third, any class function K satisfies:

K(H_f') = K(H_f).

Fourth, the eigenvalues of rho(H_f) are invariant under gauge transformation when rho is a representation.

Fifth, trace-based Wilson observables descend to the physical configuration space.

The covariance of holonomy illustrates a general pattern.

Open transport transforms at both endpoints.

Closed transport transforms only at the basepoint.

Taking a conjugacy-invariant function removes the remaining frame dependence.

This sequence converts local presentation-dependent data into a physical observable:

edge transports -> loop composite -> conjugacy-invariant loop quantity.

The same logic applies to larger closed paths.

For any loop gamma based at v:

H_gamma' = g_v o H_gamma o g_v^-1.

Thus:

W_gamma = Tr rho(H_gamma)

is gauge invariant.

Closed-loop observables probe the relational structure without requiring an absolute comparison frame.

The gauge transformation law also shows why a nontrivial holonomy cannot generally be removed by a local transformation.

Conjugation can change its representative but not its conjugacy class.

If H_f is not conjugate to the identity, no gauge transformation makes it equal to the identity.

Since the identity forms its own conjugacy class, any genuinely nonidentity holonomy remains nonidentity under gauge transformation.

Thus nontrivial curvature is not a mere artifact of local presentation.

It is encoded in the gauge-invariant conjugacy structure.

In an Abelian theory, conjugation acts trivially:

g H_f g^-1 = H_f.

The holonomy itself is gauge invariant.

In a non-Abelian theory, only its conjugacy class is invariant.

This distinction affects the types of observables and defects that can occur.

Abelian curvature can often be represented by additive flux variables.

Non-Abelian curvature retains orientation and ordering information through group multiplication.

The transformation law also controls how matter detects curvature.

Suppose a matter vector psi_v is transported around the face:

psi_v -> rho(H_f) psi_v.

Under a gauge transformation:

psi_v' = rho(g_v) psi_v,

and:

rho(H_f') psi_v'
= rho(g_v H_f g_v^-1) rho(g_v) psi_v
= rho(g_v) rho(H_f) psi_v.

Thus the physical effect of the loop is covariant.

A measurable comparison between the initial and returned matter state can be constructed using gauge-invariant inner products, traces, or interference terms.

The face holonomy therefore has operational meaning.

It describes how a local probe changes after coherent transport around a loop.

In a continuum limit, small face holonomy is expected to approximate the exponential of integrated curvature:

H_f approximately equals exp(area_f times F_f),

with ordering and representation conventions.

For a sufficiently small face and weak curvature:

H_f approximately equals id + area_f F_f + higher-order terms.

The finite model does not assume this relation as fundamental. It becomes a recovery condition when a smooth continuum interpretation is sought.

The exact finite quantity is H_f.

Curvature tensors and differential forms are approximations or derived descriptions that may emerge under refinement.

This ordering is consistent with the equivalence-first framework:

finite identity transport is primary;

infinitesimal curvature is a continuum representation of its loop obstruction.

4.4 Curvature as Failed Identity Closure

Curvature is conventionally defined through a connection and its field strength. In differential geometry, parallel transport around an infinitesimal loop fails to return a vector unchanged when curvature is present. In gauge theory, the field strength measures the noncommutativity of covariant derivatives and appears in small-loop holonomy.

The present framework retains this mathematical content while changing its foundational interpretation.

The transported entity is not merely a vector in a preassigned fiber. It is a local presentation of physical identity.

Curvature therefore means:

a local identity structure transported coherently around a closed sequence of contexts does not return by the trivial identification.

Let:

A_v

be the local identity object at the base vertex v.

After transport around a face f, the object returns to itself through:

H_f : A_v -> A_v.

When:

H_f = id_(A_v),

the transported identity closes trivially.

When:

H_f != id_(A_v),

the return identification differs from the neutral one.

The failure can be quantified by the coherence cost:

K(H_f,id).

In the simplest notation:

K(H_f)

is understood to measure the deviation of H_f from the identity.

The phrase failed identity closure should not be interpreted as logical inconsistency.

The transport maps remain well-defined and composable. The failure concerns trivial return, not existence of composition.

A nontrivial loop may be perfectly coherent as a loop while producing a nonidentity automorphism.

Thus curvature is structured nonclosure rather than breakdown of the theory.

This distinction is important.

A loop with nontrivial holonomy may encode stable physical information.

Examples include:

electromagnetic phase;

non-Abelian color rotation;

geometric rotation;

Berry phase;

topological flux;

or a defect winding.

The loop closes spatially or combinatorially but not trivially in identity space.

The theory therefore distinguishes several notions of closure:

combinatorial closure: the path returns to its starting vertex;

categorical closure: the composite is an automorphism of the starting object;

identity closure: the composite equals the identity automorphism;

higher closure: alternative composites are related by appropriate higher morphisms.

Curvature appears when combinatorial and categorical closure hold but identity closure fails.

In a higher theory, failure may itself be trivialized by a 2-morphism.

Suppose:

B_f : H_f => id_(A_v).

Then H_f may be nontrivial as a 1-morphism but equivalent to the identity at the next level.

Whether the face is considered physically curved depends on which truncation level the theory treats as observable.

This possibility shows why curvature and coherence become richer in higher gauge theory.

At the 1-groupoid level, nonidentity H_f indicates curvature.

At the 2-groupoid level, one must also ask whether a 2-equivalence trivializes it.

The present paper initially works at the 1-truncated level. Curvature is therefore determined by the holonomy class.

The coherence action converts curvature into dynamics.

A typical local term is:

S_face = beta sum_f w_f K(H_f).

Low-action configurations favor holonomies near the identity, according to the chosen class function.

This produces a tendency toward local coherent closure.

However, the minimum action need not be globally trivial.

Nontrivial topology, boundary conditions, matter coupling, or competing identity sectors may require persistent curvature.

The resulting balance can generate localized structures.

A defect may be defined as a region where the holonomy cannot be transformed to the trivial class while maintaining the required global constraints.

In this sense:

curvature is local failed closure;

defect is stable or topologically protected failed closure.

The action must determine whether curvature spreads or localizes.

If the cost is convex and no topological obstruction exists, curvature may disperse toward a flat configuration.

If a fixed total flux or winding is imposed, the minimum may distribute the curvature uniformly or concentrate it, depending on the action.

If nonlinear matter coupling is present, localized soliton-like solutions may emerge.

The concept of failed identity closure also connects gauge curvature with spacetime curvature.

In gauge theory, internal states return transformed after transport around a loop.

In gravitational geometry, vectors or frames return rotated or boosted after transport around a spacetime loop.

Both are instances of nontrivial identity transport.

The difference lies in what identity structure is being transported:

internal gauge identity;

tangent-frame identity;

spin structure;

or a more general local physical type.

A unified theory may treat these as different automorphism sectors of one broader relational object.

This remains a conjectural direction. The finite model first establishes the common transport logic without assuming that internal and spacetime curvature are already unified.

The curvature interpretation also clarifies the role of flat connections.

A flat transport field satisfies:

$H_f = id$

for every contractible elementary face.

This does not imply that every edge transport equals the identity.

Nontrivial edge variables may represent gauge choices or global topological holonomy.

A flat configuration can therefore contain rich relational structure without local curvature.

If K is simply connected and boundary conditions are trivial, a flat configuration may often be gauge transformed to:

U_e = id

on all edges of a spanning tree and, under suitable conditions, on the full complex.

If K is not simply connected, global holonomies may remain.

This provides a precise separation between:

local presentation redundancy;

local curvature;

global topology.

The phrase curvature as failed identity closure is therefore not a metaphor added to ordinary gauge mathematics. It is an interpretive reorganization of exact holonomy structure.

Its value will depend on whether the same formulation can also support:

matter defects;

mass as minimum coherence cost;

emergent geometry;

and higher identity dynamics.

4.5 Higher Coherence

The groupoid model assigns one morphism to each edge and forms exact composites along paths. This is sufficient for ordinary gauge theory, but it treats equality between alternative composites too rigidly for the full Univalent Relational program.

Suppose two paths p and q connect the same vertices:

p,q : v -> w.

The corresponding transports are:

U_p,U_q : A_v -> A_w.

At the 1-groupoid level, one asks whether:

$U_p = U_q$.

In a higher theory, the more appropriate question may be whether there exists a 2-morphism:

$\alpha : U_p \Rightarrow U_q$.

The two transports need not be literally identical. They may be coherently equivalent.

This distinction is the beginning of higher coherence.

A 2-morphism represents a transformation between transformations. It may encode:

a continuous deformation of one path into another;

a gauge transformation between gauge transformations;

a surface interpolation;

an equivalence between two composite processes;

or a witness that two local constructions implement the same physical identification.

For a face f, the boundary transport H_f may be compared with the identity by a 2-morphism:

$B_f : H_f \Rightarrow \mathrm{id}_{(A_v)}$.

If such a 2-morphism exists, the face loop is trivialized at the higher level.

The physical interpretation depends on whether B_f is:

unique;

dynamical;

observable;

costly;

or itself subject to higher equivalence.

Two trivializations:

$B_f, B_f' : H_f \Rightarrow \mathrm{id}$

may be connected by a 3-morphism.

The hierarchy can continue indefinitely.

This is the structure modeled abstractly by an infinity-groupoid and internally by higher identity types in Homotopy Type Theory.

The higher extension is necessary for several reasons.

First, strict equality is often too restrictive for physical composition.

Two experimental procedures may differ microscopically while being physically equivalent.

Two coordinate transformations may be related by a smooth deformation.

Two decompositions of a process may produce canonically equivalent but not literally identical composites.

A higher morphism records the equivalence without forcing literal identity.

Second, local-to-global gluing naturally produces higher coherence conditions.

Suppose local presentations A_i are defined on overlapping regions.

On pairwise overlaps, one has transition morphisms:

u_ij : A_i -> A_j.

On triple overlaps, the two composites:

u_jk o u_ij

and:

u_ik

may be related by a 2-morphism:

alpha_ijk : u_jk o u_ij => u_ik.

On quadruple overlaps, the different ways of composing the alpha_ijk must satisfy a 3-dimensional coherence condition.

The global object is determined not only by local presentations and pairwise equivalences, but by the full hierarchy of compatibility witnesses.

Third, higher gauge fields require surface and volume transport.

Ordinary connections assign transport to paths.

A 2-connection also assigns transport to surfaces.

Higher connections assign data to higher-dimensional regions.

These structures are natural for strings, membranes, gerbes, higher-form gauge fields, and generalized fluxes.

Fourth, anomalies can be interpreted as failures of higher coherence.

A set of local transformations may compose consistently pairwise but fail globally at a higher level.

Such failure may appear as:

an associativity obstruction;

a projective phase;

a cocycle;

a nontrivial higher cohomology class;

or an incompatibility in quantization.

Higher coherence is therefore not merely an optional refinement. It may carry the obstruction data determining whether a theory is globally consistent.

Fifth, quantum interference may depend on higher relations among histories.

If two transport paths are alternative identifications between the same boundary contexts, a 2-morphism can represent their deformation relation.

Distinct higher-homotopy classes may contribute different phases or amplitudes.

This suggests that interference may be organized by the topology of the space of identifications, not only by a set of classical trajectories.

Sixth, measurement as truncation requires a hierarchy to truncate.

A 0-truncation retains connected components while forgetting nontrivial paths.

A 1-truncation retains ordinary equivalences but forgets higher equivalences between them.

Different observational regimes may preserve different levels of the relational structure.

The groupoid model represents a 1-truncated physical world.

The full theory may require a higher object whose accessible image depends on the measurement process.

Despite these motivations, the present constructive paper should not place the full higher structure into the initial computational model.

The first task is to solve the 1-groupoid theory completely enough to determine:

its gauge invariants;

its defect sectors;

its partition function;

its continuum behavior;

and its non-inserted predictions.

Higher data should then be introduced in controlled extensions.

A first finite higher model may use a strict 2-group or crossed module.

A crossed module consists of groups H and G, a homomorphism:

partial : H -> G,

and an action of G on H satisfying compatibility conditions.

One may assign:

G-valued transport to edges;

H-valued transport to faces.

The boundary of the face variable is related to the edge holonomy.

This permits a discrete formulation of higher curvature and surface holonomy.

Alternatively, one may use a finite 2-groupoid in which:

vertices carry objects;

edges carry 1-morphisms;

faces carry 2-morphisms.

The face variable then compares alternative boundary composites.

The choice between strict and weak higher structures should be made according to computational tractability and the physical question.

Strict models are easier to enumerate but may suppress physically meaningful associators.

Weak models are more general but require additional coherence data.

A staged program is therefore appropriate:

Stage 1: finite groupoid gauge dynamics;

Stage 2: face 2-morphisms and higher flatness;

Stage 3: higher-form charges and surface observables;

Stage 4: continuum cohesive interpretation;

Stage 5: full HoTT formalization.

At every stage, higher structure must earn its place by producing a calculable consequence.

Possible success criteria include:

new conserved higher charges;

additional stable defect classes;

modified partition functions;

surface-dependent interference;

higher anomaly constraints;

or selection rules absent from the 1-groupoid model.

The full conceptual hierarchy can be stated as:

edge transport compares local identities;

face holonomy compares path composites;

2-morphisms compare holonomies or transports;

3-morphisms compare the 2-comparisons;

higher morphisms enforce coherence of the complete relational structure.

The central lesson is that physical sameness may itself have internal levels.

Two local presentations may be equivalent.

Two equivalences may be equivalent.

Two such higher equivalences may also be equivalent.

A complete equivalence-first theory must eventually determine which levels possess physical content, which are gauge redundancy, which are observable, and which are erased by finite measurement.

The present section establishes the first level with precision.

Path transport defines persistence through composition.

Face holonomy detects nontrivial return.

Gauge covariance ensures that physical loop data are presentation-independent.

Curvature is the failure of identity transport to close trivially.

Higher coherence generalizes literal equality of transports to structured equivalence among them.

These constructions provide the geometric core required for the coherence action developed in the next section.

5. The Univalent Relational Coherence Action

5.1 Requirements on the Action

The mathematical arena specifies which configurations are possible, which local transformations count as changes of presentation, and which loop quantities survive gauge transformation. It does not yet determine which configurations are physically realized.

A dynamical theory requires an action or equivalent selection principle.

Let:

Phi = ({A_v},{U_e},{psi_v})

denote a field configuration on the finite oriented 2-complex K.

The Univalent Relational coherence action assigns a real number:

S_URD(Phi)

to each admissible configuration.

In the Euclidean formulation developed first, configurations with smaller action receive greater statistical weight:

weight(Phi) = exp[-S_URD(Phi)/hbar].

In a classical limit, physically preferred configurations are stationary points or minima of the action subject to boundary conditions and topological constraints.

The action must satisfy several requirements if it is to represent more than a formal analogy.

The first requirement is gauge invariance.

For every admissible local change of presentation g:

S_URD(g · Phi) = S_URD(Phi).

This condition ensures that the action depends on physical relational content rather than arbitrary local frames.

The second requirement is locality.

The action should be expressible, at least initially, as a sum of terms associated with vertices, edges, faces, and boundaries:

S_URD = sum_v S_v + sum_e S_e + sum_f S_f + S_boundary.

Locality makes the model computationally tractable and permits a controlled continuum limit.

It also reflects the idea that global physical structure arises through the coherent composition of local relations.

The third requirement is compositional compatibility.

If two subcomplexes are joined along a shared boundary, the action of the combined system should be determined by the actions of the parts together with explicit interface terms.

The theory should not assign unrelated values to a process and its decomposition.

The fourth requirement is boundedness in the Euclidean formulation.

For finite statistical sums and stable minimization, the principal Euclidean terms should satisfy:

S_URD(Phi) >= S_min

for some finite lower bound.

It is convenient to normalize:

S_min = 0

when possible.

The fifth requirement is sensitivity to failed closure.

The action must distinguish a locally flat configuration from one with nontrivial face holonomy.

A configuration satisfying:

H_f = id

for every contractible face should minimize the pure local coherence term unless boundary conditions, topology, matter coupling, or sector costs require otherwise.

The sixth requirement is compatibility with nontrivial topology.

The action must not force every physically allowed configuration into one globally trivial state.

Locally flat but globally nontrivial sectors should remain possible on complexes with noncontractible loops.

Stable defects should also remain possible when topology or boundary conditions prevent complete trivialization.

The seventh requirement is representation independence.

The value assigned to a physical configuration must not depend on:

the choice of gauge;

the arbitrary labeling of vertices or groupoid objects;

the chosen basepoint of a face;

the orientation convention when orientation is not physically distinguished;

or an equivalent presentation of the same defect sector.

The eighth requirement is dimensional consistency.

Every term contributing to the action must possess the same physical dimensions.

In the initial finite model, the action may be treated as dimensionless after division by hbar or after choosing natural units. However, any continuum interpretation must restore explicit dimensions.

If edge lengths, face areas, or vertex volumes are introduced, their scaling must be specified.

The ninth requirement is recovery of established gauge dynamics.

When the identity groupoid is reduced to one object A and the face cost is chosen appropriately, the action must reduce to the Wilson lattice gauge action.

This recovery condition constrains the allowed form of the coherence term.

The tenth requirement is extensibility.

The action should permit the later inclusion of:

matter fields;

multiple identity sectors;

higher morphisms;

fermionic variables;

causal structure;

and dynamical geometry.

The minimal form should not preclude these extensions.

The eleventh requirement is exact finite computability.

For a finite complex and finite groupoid, one should be able to enumerate all allowed configurations or sample them systematically, evaluate the action, classify gauge orbits, and compute the partition function.

The twelfth requirement is physical nontriviality.

The action should be capable of producing more than a preference for universal flatness.

It must permit competition among:

local coherence;

global topology;

matter variation;

identity-sector cost;

boundary forcing;

and defect stability.

Without such competition, the theory would collapse into a trivial model in which every transport is gauge equivalent to the identity.

The action is therefore built from several components rather than one universal penalty.

The general form is:

S_URD
= S_face

- S_matter
- S_identity

- S_boundary
- S_defect
- optional higher terms.

Each term has a distinct role.

S_face measures local failure of identity closure.

S_matter measures incompatibility between transported and actual local matter data.

S_identity assigns cost to the occupation or maintenance of local identity types.

S_boundary specifies externally constrained relational data.

S_defect distinguishes or stabilizes nontrivial topological sectors.

Higher terms may later measure coherence among face data, higher-form curvature, or long-range relational constraints.

The action should be understood as a hypothesis about physical selection.

The mathematical arena lists possible relational configurations.

The action ranks them.

The equivalence-first proposal is that the ranking is fundamentally a ranking of coherent identity structures rather than a ranking of isolated states.

This does not imply that the universe always minimizes local difference.

Physical organization requires distinction as well as equivalence.

A completely trivial configuration may possess maximal local closure but no localized matter, no nontrivial topology, no charge, and no complex structure.

The physically relevant minimum may therefore occur under constraints that prevent complete trivialization.

The action must encode the balance between:

coherence sufficient for persistence;

and obstruction sufficient for structure.

This balance is the dynamical core of Univalent Relational Gauge Dynamics.

5.2 Face Coherence Term

The most basic dynamical quantity is the cost assigned to face holonomy.

For each oriented face f, choose a base vertex v_f and define:

H_f in Aut(A_(v_f)).

A face-coherence function is a map:

K_A : Aut(A) -> R

for each identity object A.

The local face term is:

S_f = beta w_f K_(A_(v_f))(H_f),

where:

beta >= 0

is a coupling parameter and:

w_f > 0

is an optional face weight.

The complete face action is:

S_face = beta sum over f in F of w_f K_(A_(v_f))(H_f).

The function K must satisfy several conditions.

First:

K_A(id_A) = 0.

Trivial identity closure carries no local curvature cost.

Second:

K_A(h) >= 0

in the Euclidean model.

Third, K must be conjugacy invariant:

K_A(g h g^-1) = K_A(h)

for every g,h in Aut(A).

This ensures gauge invariance.

Fourth, when orientation is conventional rather than physical:

K_A(h^-1) = K_A(h).

This ensures that reversing the face orientation does not alter its cost.

Fifth, equivalent identity objects should carry compatible face costs.

Suppose:

u : A -> B

is an admissible groupoid morphism.

Then the automorphism h of A is transported to:

u h u^-1 in Aut(B).

Compatibility requires:

K_B(u h u^-1) = K_A(h).

Thus the face cost belongs to the physical identity sector rather than to one arbitrary object representative.

For a finite group, K may be specified as a table of nonnegative values constant on conjugacy classes.

For a compact Lie group with a unitary representation rho, a standard choice is:

K(h) = c Re Tr[I - rho(h)],

where c is a normalization factor.

This function is minimized at the identity in a faithful representation, although some nonidentity central elements may require careful treatment depending on rho.

For the group U(1), write:

h = exp(i theta).

A natural choice is:

K(h) = 1 - cos(theta).

For small theta:

K(h) approximately equals theta^2 / 2.

Thus the local cost is quadratic in weak Abelian curvature.

For a finite cyclic group Z_n, with:

h = exp(2 pi i k / n),

the same expression gives:

K(k) = 1 - cos(2 pi k / n).

The distinct flux values then carry discrete costs.

For a non-Abelian finite group, K can depend on the conjugacy class of h.

This permits different types of nontrivial holonomy to carry different energetic penalties.

The parameter beta controls the stiffness of identity closure.

Large beta strongly suppresses nontrivial face holonomy.

Small beta permits substantial local curvature fluctuations.

In statistical language, beta often behaves like an inverse coupling strength, though the exact physical interpretation depends on normalization.

The face weights w_f may encode:

face area;

local coupling variation;

inhomogeneous coherence stiffness;

orientation multiplicity;

or geometric data emerging from another sector.

If the complex is regular and no local distinction is intended, one may take:

w_f = 1.

The initial model should begin with uniform weights to avoid inserting unnecessary geometric structure.

The face term possesses an important interpretive duality.

Mathematically, it is a class function of holonomy.

Physically, it is the cost of nontrivial identity return.

The theory does not introduce a separate curvature variable at the outset. Curvature is represented exactly by the loop transport.

The action asks how costly it is for an identity transported around an elementary relational loop to return through an automorphism h rather than through id.

This interpretation becomes especially clear in the weak-curvature limit.

Suppose the group is continuous and:

H_f = exp(X_f),

where X_f lies near zero in the Lie algebra.

For a suitable class function:

K(H_f) approximately equals c <X_f,X_f>.

If X_f is proportional to the integrated field strength over the face, then:

S_face

approaches a discrete version of the Yang-Mills curvature-squared action.

The conventional continuum field strength therefore emerges as the infinitesimal representation of finite failed identity closure.

The face action can support several regimes.

In the flat regime, all contractible face holonomies are the identity.

In the weak-curvature regime, H_f remains near the identity and continuum approximation may be valid.

In the strong-fluctuation regime, many conjugacy classes contribute substantially.

In a topologically constrained regime, local curvature may be required to satisfy a fixed global flux or winding condition.

In a defect regime, nontrivial holonomy may be concentrated in a localized region.

The form of K influences which regime is preferred.

A strictly convex K near the identity favors distributed weak curvature under some constraints.

A nonconvex K may permit several preferred holonomy classes and support phase structure.

A K with degenerate minima may allow multiple locally coherent sectors.

Such degeneracy can be used to model spontaneous selection among different relational phases.

However, degeneracy should not be introduced merely to force desired outcomes.

The simplest class function should be tested first.

The face action also determines whether local curvature can be interpreted as a defect.

A single nontrivial face in an otherwise flat complex may carry a finite action:

Delta S = beta w_f K(H_f).

But this alone does not guarantee stability.

The holonomy may be removable by redistributing edge transports or changing the surrounding configuration.

A genuine defect requires a topological, boundary, or dynamical obstruction preventing continuous or gauge-equivalent removal.

The face action provides the energetic cost, while the global structure provides stability.

A distinction must therefore be maintained among:

curvature excitation;

topological defect;

and particle-like stable sector.

Every defect contains nontrivial relational structure, but not every local curvature fluctuation is a defect.

The face action is the first component of the theory because it captures the most direct expression of failed coherence. It is not the whole theory because physical structure also depends on how matter and identity types are transported through that curvature.

5.3 Transport and Matter Term

Matter data introduce local degrees of freedom that transform under the automorphism structure of the identity objects.

For each vertex v, let:

psi_v in V_(A_v),

where V_(A_v) is the representation space associated with the identity object A_v.

For each groupoid morphism:

u : A -> B,

the representation functor assigns:

rho(u) : V_A -> V_B.

The edge transport:

U_e : A_s(e) -> A_t(e)

therefore induces:

rho(U_e) : V_(A_s(e)) -> V_(A_t(e)).

The actual matter value at the target can be compared with the transported matter value from the source.

Define the covariant edge difference:

D_e psi
= psi_t(e) - rho(U_e) psi_s(e).

This expression lies in the target representation space V_(A_t(e)).

The matter-transport action is:

S_transport
= kappa sum over e in E_plus of w_e ||D_e psi||^2,

where:

kappa >= 0

is the transport coupling,

w_e > 0

is an optional edge weight, and E_plus contains one selected orientation for each unoriented edge to avoid double counting.

The norm must be invariant under local automorphisms:

||rho(g)x|| = ||x||

for every admissible g.

Under a gauge transformation:

D_e psi -> rho(g_t(e)) D_e psi.

Therefore:

||D_e psi||^2

is gauge invariant.

The term is minimized when:

psi_t(e) = rho(U_e) psi_s(e).

This condition means that the matter field is perfectly compatible with the identity transport along the edge.

The transported presentation and the actual target presentation agree.

A nonzero D_e psi measures relational mismatch.

The matter action therefore has an equivalence-first interpretation:

matter variation is not measured by subtracting isolated values in unrelated frames;

it is measured after one value has been transported into the frame of the other.

This is the finite categorical analogue of the covariant derivative.

A local potential may also be introduced:

S_potential = sum over v in V of w_v V_(A_v)(psi_v).

Gauge invariance requires:

V_A(rho(g) psi) = V_A(psi).

The complete matter term becomes:

S_matter
= kappa sum over e in E_plus of w_e ||psi_t(e) - rho(U_e) psi_s(e)||^2

- sum over v in V of w_v V_(A_v)(psi_v).

The potential may control:

local amplitude;

symmetry breaking;

self-interaction;

vacuum expectation values;

or occupation of different representation sectors.

For a complex scalar field in a U(1) representation, one might choose:

V(psi) = m_0^2 |psi|^2 + lambda |psi|^4.

However, the parameter m_0 should not immediately be identified with the final physical mass proposed by the theory.

It is a local coefficient in the matter potential.

The equivalence-inertia hypothesis concerns the minimum total action of a stable physical defect after gauge redundancy and presentation dependence have been removed.

A bare local parameter and an emergent defect mass are conceptually different.

The theory should avoid inserting the desired particle mass directly through V and then claiming to have derived it.

For the first gauge-only experiments, one may set:

psi_v = 0

or omit matter entirely.

For the first matter-coupled experiments, the simplest representation should be used.

Possible choices include:

a one-dimensional representation of a finite Abelian group;

a low-dimensional unitary representation of a finite non-Abelian group;

or a permutation representation arising naturally from a groupoid action.

The matter term permits gauge holonomy to produce observable effects.

Transporting psi around a closed loop gives:

psi_v -> rho(H_f) psi_v.

If rho(H_f) differs from the identity action, the matter field detects the loop curvature.

This provides an operational meaning for holonomy.

The field does not observe the raw edge variables individually. It observes the accumulated transformation produced by relational transport.

The coupling between matter and holonomy can generate frustration.

Suppose the potential favors the same local matter orientation at every vertex, but nontrivial holonomy prevents that orientation from being transported consistently around a loop.

Then no configuration can minimize every edge term simultaneously.

The resulting incompatibility may force:

localized zeros of psi;

phase winding;

domain walls;

vortices;

or other defect structures.

Thus matter defects can arise from the competition between:

local potential preference;

transport coherence;

and global holonomy.

This mechanism is promising for the equivalence-first interpretation of particles.

A stable excitation need not be inserted as a primitive object.

It may appear because the relational constraints cannot be satisfied everywhere at once.

The transport term can also support spontaneous symmetry breaking.

If the local potential has a set of minima invariant under a group G but a particular configuration selects one minimum, the stabilizer of psi may be a subgroup H of G.

The effective symmetry is then reduced from G to H.

Within the groupoid interpretation, the matter configuration reduces the automorphism structure that leaves the full field configuration invariant.

This provides a standard mechanism of symmetry breaking inside the broader equivalence-first architecture.

The theory must still distinguish ordinary symmetry breaking from the stronger proposal of gauge-group selection.

Symmetry breaking begins with a specified automorphism group and selects a subgroup through a field configuration.

Gauge-group selection would require competition among different identity objects or groupoid sectors whose automorphism groups were not uniquely fixed at the outset.

The matter term may contribute to that competition by making some sectors more compatible with low-cost transport than others.

The representation functor rho therefore plays a crucial role.

It determines:

how identity automorphisms act on matter;

which charges are possible;

which covariant differences can be formed;

which holonomies are observable;

and which symmetry-breaking patterns can occur.

A representation should not be chosen arbitrarily if the theory aims eventually to derive matter content.

In the first constructive model, however, rho is an explicit input.

The paper should state this clearly.

The initial goal is to determine whether the equivalence framework can reproduce correct gauge-matter coupling and support stable relational defects.

Derivation of the observed representation content is a later objective.

Fermionic matter requires additional structure.

Grassmann variables, spin structures, orientation data, and an appropriate discrete Dirac operator would have to be introduced.

The current scalar or finite-dimensional matter sector is not yet sufficient to reproduce the Standard Model.

The constructive sequence should be:

pure transport;

bosonic matter representation;

topological defects;

fermionic extension;

chiral structure;

and only then realistic particle content.

The transport and matter term establishes the first bridge from identity coherence to local excitation.

It says that matter is physically coherent when its local presentations agree under the relational transport field.

Deviation from that agreement carries action cost.

When global consistency is impossible, the mismatch may become localized and persistent.

This is the mechanism through which a coherence field can begin to generate matter-like structure.

5.4 Identity-Selection Term

A conventional gauge theory fixes one gauge group and one local fiber type before dynamics begins.

A groupoid-valued theory can permit several local identity objects.

The action must then determine the cost of assigning those objects to vertices.

Let:

A_v in Ob(G)

be the identity object at vertex v.

Define a local identity-maintenance function:

M : Ob(G) -> R.

The identity-selection term is:

S_identity = sum over v in V of w_v M(A_v).

At the most elementary level, M assigns a baseline cost to occupying an identity sector.

This cost may represent:

internal coherence maintenance;

local structural complexity;

vacuum energy associated with the sector;

compatibility with surrounding transport;

or an effective chemical potential for object type.

The notation M is used cautiously.

It should not automatically be interpreted as observed mass.

It is a local action contribution associated with an identity object.

Observed particle mass, if it emerges, should be defined from the total minimum action of a stable localized sector relative to the vacuum.

If two groupoid objects A and B are physically equivalent through an admissible morphism:

u : A -> B,

then their intrinsic identity cost should ordinarily agree:

M(A) = M(B).

Otherwise the action would assign different energies to two presentations declared physically equivalent.

Thus M should be constant on connected components of the physical groupoid unless the morphism u changes additional physical structure not included in the bare object label.

This requirement is another form of observable descent.

If M varies among isomorphic representatives only because of labeling, it is not physical.

A nontrivial identity-selection term therefore becomes most meaningful when the groupoid contains several physically distinct components or when the objects carry additional structure beyond bare equivalence.

Suppose the groupoid contains sectors:

C_1, C_2, ..., C_r.

Each sector may possess a characteristic cost:

M_i.

The action can then favor one sector over another.

However, a purely local M would select the lowest-cost sector independently at every vertex and might produce a trivial uniform phase.

To support nontrivial domain structure, one may introduce an adjacency compatibility term.

Let:

J(A_v,A_w,U_e)

measure the cost of placing identity types A_v and A_w across edge e with transport U_e.

For an allowed groupoid morphism, this cost may be small.

For incompatible or weakly connected sectors, it may be large.

One may define:

S_object-edge
= eta sum over e in E_plus of J(A_s(e),A_t(e),U_e).

In the strict reversible model, U_e exists only when A_s(e) and A_t(e) lie in the same connected component.

Then J may measure variation among equivalent object presentations.

In a larger model permitting domain walls or noninvertible interfaces, J can penalize transitions between distinct sectors.

The identity-selection action becomes:

S_identity
= sum_v w_v M(A_v)

- eta sum_e J(A_s(e),A_t(e),U_e).

This creates competition among:

local sector preference;

interface cost;

transport compatibility;

and entropy of available presentations.

At finite temperature or in the Euclidean partition function, a sector with higher local cost may still dominate if it has much larger configuration multiplicity.

Thus effective identity selection depends on free energy rather than action alone.

For sector i:

F_i = E_i - T S_i,

or in the finite path-integral language:

$F_i = -k_B T \log Z_i$.

This permits an entropic form of gauge selection.

A sector with a large automorphism group may possess many transport configurations, but automorphism weighting also reduces overcounting.

The final statistical preference depends on the full moduli-groupoid structure.

This is a key point.

The theory should not assume that greater symmetry automatically implies greater probability.

Large symmetry can both increase structural possibilities and enlarge stabilizer corrections.

The competition must be calculated.

A heterogeneous identity groupoid can model several phenomena.

Symmetry enhancement occurs when the dominant sector has a larger automorphism group.

Symmetry reduction occurs when a lower-automorphism object or matter stabilizer is favored.

Phase coexistence occurs when several sectors have comparable free energies.

Domain walls occur when different sectors dominate in different regions.

Identity conversion requires noninvertible or extended processes connecting sectors not related by reversible equivalence.

The identity-selection term therefore supplies a possible route toward explaining why one gauge structure appears at low energy.

But the route remains incomplete.

If the candidate groupoid is constructed to contain only the observed gauge sector, nothing has been derived.

If the costs M and J are tuned to force the desired group to dominate, the selection is also inserted.

A meaningful result requires:

a broad but principled candidate groupoid;

simple universal cost rules;

and a preferred sector emerging robustly over a range of parameters.

The paper should therefore begin with modest finite examples.

One example might contain two object sectors:

A with Aut(A) = Z_2;

B with Aut(B) = Z_3.

A common rule for K, M, and transport multiplicity can then determine which sector dominates under different couplings.

Another example might contain objects with:

Aut(A) = Z_2,

Aut(B) = S_3,

or other small finite groups.

The purpose is not to model the Standard Model immediately.

It is to test whether dynamic automorphism selection is mathematically meaningful and computationally nontrivial.

The identity-selection term also connects to mass as coherence inertia.

A local object may carry an internal maintenance cost M(A), but a stable particle-like defect may require an extended arrangement of objects, transports, matter fields, and holonomies.

Its physical rest energy is not the sum of arbitrary object labels alone.

It is the least total excess action required to sustain the complete invariant pattern.

Thus:

local identity cost contributes to mass;

but mass is a property of the minimized physical sector.

This distinction prevents the theory from reducing to a relabeling of conventional mass terms.

5.5 Defect and Boundary Terms

Boundary conditions and topological constraints determine which configurations can be globally trivialized.

They are therefore essential to the existence and classification of defects.

Let:

partial K

denote the boundary subcomplex of K.

A boundary condition may fix or constrain:

identity objects on boundary vertices;

transport morphisms on boundary edges;

matter fields on boundary vertices;

holonomy around boundary cycles;

or the total topological sector.

The boundary action may be written generally as:

S_boundary
= B({A_v},{U_e},{psi_v} on partial K).

Several common forms are possible.

A fixed Dirichlet-type condition may prescribe:

psi_v = psi_v^(0)

on selected boundary vertices.

A fixed transport condition may prescribe:

U_e = U_e^(0)

on selected boundary edges.

A soft boundary condition may penalize deviation:

S_boundary
= lambda_boundary sum over boundary edges of d(U_e,U_e^(0))^2

or:

S_boundary
= lambda_boundary sum over boundary vertices of ||psi_v - psi_v^(0)||^2.

A boundary holonomy condition may fix:

H_gamma = h_boundary

for a cycle gamma on the boundary.

Such a condition can force nontrivial flux or winding into the interior.

Boundary transformations must be treated carefully.

If gauge transformations are required to equal the identity on the boundary, boundary values are physical external data.

If nontrivial boundary transformations are allowed, then only gauge-invariant boundary quantities are fixed.

If transformations act nontrivially at the boundary, they may generate physical boundary charges or edge modes.

The theory must state which convention is used.

For the first finite model, the simplest choice is:

g_v = id

on fixed boundary vertices.

This avoids ambiguity and makes the imposed relational data explicit.

A defect term is conceptually different from a boundary term.

The boundary term specifies external conditions.

The defect term distinguishes internal nontrivial sectors.

Let D(Phi) denote a gauge-invariant defect label.

Examples include:

a conjugacy class of global holonomy;

an element of a homotopy or cohomology class;

a winding number;

a flux value;

a domain-wall number;

or another obstruction invariant.

One may restrict the partition function to a fixed sector:

D(Phi) = d.

Alternatively, one may add a sector-dependent action:

S_defect = mu Q(D(Phi)),

where Q assigns cost or weighting to the defect label.

However, care is required.

If the defect is supposed to emerge dynamically, inserting an arbitrary stabilizing term may prejudge the result.

The first question should be whether stable defect sectors already arise from:

the face action;

matter transport;

local potential;

identity costs;

topology;

and boundary conditions.

An explicit S_defect should be introduced only when it represents a well-defined physical coupling or constraint not otherwise captured.

There are several ways defects may arise without an added defect potential.

First, global topology may prevent trivialization.

On a complex with noncontractible cycles, flat connections may have nontrivial global holonomy.

Second, boundary conditions may impose a nonzero total flux.

Third, the matter field may possess a vacuum manifold with nontrivial topology.

A winding of psi around a boundary can force a zero or singular core in the interior.

Fourth, heterogeneous identity sectors may create domain walls.

Fifth, higher coherence conditions may admit nontrivial cocycles.

The defect energy is then the excess action relative to the vacuum sector:

Delta S_D
= inf over Phi with D(Phi)=D of S_URD(Phi)

- inf over Phi with D(Phi)=0 of S_URD(Phi).

The proposed rest energy is:

m_D c^2 = Delta E_D,

after the Euclidean action has been related to a physical energy through a specified temporal or transfer-matrix interpretation.

This qualification is important.

A dimensionless Euclidean action difference is not automatically a mass.

To extract mass, one generally needs:

a temporal direction;

correlation-function decay;

a transfer matrix;

or a static energy functional.

In a purely spatial finite model, Delta S_D is initially a coherence-cost gap.

It becomes a mass candidate only after the dynamical interpretation is completed.

The paper should distinguish:

coherence action gap;

static defect energy;

and relativistic rest mass.

Premature identification would overstate the result.

A more rigorous lattice definition of mass may use correlation functions.

Let O_D create or detect a defect-sector excitation.

If the Euclidean two-point function satisfies:

<O_D(t) O_D(0)> approximately proportional to exp(-m_D t)

at large separation t, then m_D is the mass gap associated with that excitation in lattice units.

This is closer to standard field-theoretic practice.

The equivalence-inertia interpretation would then state that the extracted mass arises from the least action cost of propagating the defect's identity coherently through Euclidean time.

The boundary and defect sectors are therefore where the framework can move from kinematics to physically distinguishable structure.

Gauge transformations remove changes of presentation.

Defect labels remain because they cannot be removed by admissible local equivalence.

This gives a sharp conceptual contrast:

gauge variation = removable descriptive difference;

defect charge = nonremovable relational obstruction.

The theory should test whether defect labels are:

gauge invariant;

stable under local perturbations;

quantized;

composable;

and preserved or transformed under interactions.

Only then can they be considered candidates for physical charge or particle identity.

5.6 Complete Finite Action

Combining the preceding terms gives the initial Euclidean action:

S_URD(Phi)
= beta sum over f in F of w_f K(H_f)

- kappa sum over e in E_plus of w_e ||psi_t(e) - rho(U_e) psi_s(e)||^2
- sum over v in V of w_v V_(A_v)(psi_v)
- sum over v in V of w_v M(A_v)
- eta sum over e in E_plus of J(A_s(e),A_t(e),U_e)
- S_boundary(Phi)
- S_defect(Phi).

Each component has a distinct interpretation.

The face term measures failure of local identity closure.

The edge matter term measures failure of local matter presentations to agree under transport.

The potential selects locally preferred matter configurations.

The identity term assigns baseline cost to local identity sectors.

The object-edge term measures compatibility among neighboring identity types.

The boundary term specifies external physical context.

The defect term, when present, weights a gauge-invariant global obstruction.

The simplest pure gauge model sets:

psi_v absent,

M(A_v) constant,

J = 0,

S_defect = 0,

and fixes one identity object A at every vertex.

Then:

S_URD = beta sum_f K(H_f) + S_boundary.

In the one-object case with a Wilson class function, this becomes ordinary lattice gauge theory.

The next level permits multiple equivalent object presentations while retaining one connected groupoid component.

The next permits several disconnected sectors.

Matter is then added through rho and V.

This sequence allows each new physical ingredient to be isolated.

The total action must be tested for gauge invariance term by term.

The face term is invariant because K is conjugacy invariant.

The matter transport term is invariant because D_e psi transforms covariantly and the norm is invariant.

The local potential is invariant when V is constant on automorphism orbits.

The identity cost is invariant when M is constant on physically equivalent object presentations.

The compatibility term is invariant when:

J(A,B,U)
= J(A,B,g_B U g_A^-1).

The boundary term is invariant under the permitted boundary gauge transformations.

The defect term is invariant when D(Phi) is a physical sector label.

If any term fails these requirements, the total action does not descend to the moduli groupoid.

The stationary equations of the finite action are discrete analogues of field equations.

Variation or local update of U_e affects every face containing e and every matter transport term on e.

Variation of psi_v affects the adjacent covariant differences and the local potential.

Variation of A_v, when object types are dynamical, changes the permitted incident morphisms, automorphism group, representation space, and identity cost.

For finite groups and discrete object sets, variation is replaced by comparison among allowed assignments.

For Lie groups and continuous matter spaces, ordinary differentiation may be used.

A local edge update seeks a new U_e minimizing the combined mismatch among:

neighboring face closures;

and matter transport.

A local matter update seeks a new psi_v balancing:

agreement with transported neighbors;

and the local potential.

An object update seeks a new A_v balancing:

identity cost;

available morphisms;

automorphism structure;

matter compatibility;

and interface penalties.

This gives the action an interpretation as a coupled coherence optimization.

The model searches for configurations in which:

loops close as coherently as possible;

matter transports consistently;

identity sectors fit their neighborhoods;

and nontrivial constraints are accommodated at minimum cost.

The action can also be expressed probabilistically.

The finite Euclidean partition function is:

Z = sum over physical configurations [Phi] of
exp[-S_URD(Phi)/hbar] / |Aut(Phi)|.

Equivalently, under suitable finite conditions:

Z = (1 / |Gauge|) sum over raw Phi of exp[-S_URD(Phi)/hbar].

The first form emphasizes the moduli groupoid.

The second may be convenient computationally when the gauge group is uniform and finite.

Expectation values are:

The theory should compute both forms in small examples and verify their agreement.

This provides a strong test of correct orbit and stabilizer handling.

The action also supports Monte Carlo simulation for larger finite systems.

Local updates must preserve admissibility.

Possible update moves include:

changing one edge transport;

changing one vertex matter value;

changing one object type together with compatible incident morphisms;

inserting or moving a defect;

or applying a gauge transformation as a null test.

A gauge transformation should leave the action unchanged and therefore be accepted with unit probability in a gauge-redundant raw simulation.

Physical updates may change the action and be accepted according to the chosen sampling algorithm.

The complete finite action is not yet unique.

Different choices of K, V, M, and J define different models.

The theory therefore requires a principle for selecting simple forms.

The initial choices should satisfy:

minimal parameter count;

gauge invariance;

known continuum behavior in the one-object limit;

and sufficient structure to test defect stability and sector selection.

The first paper should not explore an uncontrolled family of arbitrary actions.

A recommended starting model is:

one finite group or small finite groupoid;

uniform weights;

Wilson-like K;

a simple quadratic-plus-quartic matter potential if matter is included;

no explicit defect term;

and fixed or periodic boundaries.

Only after its behavior is understood should more elaborate coherence costs be introduced.

The scientific question is not whether some complicated action can be tuned to generate interesting patterns.

It is whether a simple equivalence-first action produces robust physical structure.

5.7 Principle of Maximal Coherent Equivalence

The action can be summarized by a broader variational principle:

physically realized structures maximize coherent equivalence subject to the costs and constraints required to preserve nontrivial physical distinction.

This principle must be stated carefully.

It does not say that every object should become equivalent to every other object.

It does not say that the universe seeks featureless uniformity.

It does not say that curvature, defects, or differences are errors.

Equivalence is always relative to a specified physical structure and class of admissible transformations.

A coherent system preserves identity where preservation is physically possible.

It also retains obstructions where topology, boundary conditions, matter content, or dynamical constraints prevent complete identification.

The action represents this balance.

The face term favors trivial return around local loops.

The matter term favors agreement under transport.

The identity term favors low-cost local structures.

The boundary and defect terms preserve imposed or emergent distinctions.

The result may be a globally organized configuration containing local nontriviality.

This principle resembles least-action reasoning but changes the object of minimization.

The primitive variables are not interpreted merely as fields living on spacetime.

They are maps of identity transport and local presentation.

The action measures how costly it is for the relational system to sustain a coherent pattern.

The classical equations therefore describe the extremization of coherence under constraint.

Several limiting cases clarify the principle.

If the complex is simply connected, the boundary is trivial, matter is absent, and K has a unique minimum at the identity, the global minimum may be a flat configuration gauge equivalent to trivial transport.

In this case, maximal coherent equivalence produces no defect.

If the complex has noncontractible cycles, locally flat configurations may retain global holonomy.

The system preserves maximal local coherence while retaining topological distinction.

If the boundary fixes nontrivial flux, the interior cannot become globally trivial.

The minimum distributes or localizes the required obstruction at least cost.

If matter has a nontrivial vacuum manifold, winding may force a localized core.

The defect is the minimum-cost solution compatible with global matter identity.

If several object sectors compete, the preferred phase balances local cost, transport compatibility, and moduli-space entropy.

The dominant automorphism structure may then emerge dynamically.

Thus maximal coherent equivalence does not erase structure.

It identifies the most economical way for structure to persist.

The principle can be contrasted with universal symmetry.

Universal symmetry would treat all differences as transformations of one homogeneous substrate.

Maximal coherent equivalence permits disconnected sectors and irreversible transitions.

It does not presume that every distinction is gauge.

The admissibility selector determines which differences are removable.

The action determines how the removable and nonremovable structures arrange themselves.

The principle also clarifies the relation between persistence and mass.

A stable defect persists because removing it would require leaving its topological sector, violating boundary conditions, or crossing an action barrier.

Its inertia reflects the cost of transporting and maintaining the relational pattern.

If the minimum static or propagation cost can be calculated and shown to correspond to a mass gap, the phrase mass as equivalence inertia gains quantitative meaning.

The principle further suggests a route to geometry.

If coherent transport between two contexts carries a minimum cost, then low-cost relations define proximity.

A network of such costs may produce an effective metric.

Curvature then records path-dependent identity transport.

Matter defects alter the surrounding coherence field.

In a later continuum limit, the same action might couple effective matter and geometry.

This possibility remains conjectural at the present stage.

The gauge model must first succeed on its own terms.

The maximal-coherence principle is scientifically useful only if it produces definite mathematical consequences.

These include:

gauge-invariant stationary equations;

stable defect classes;

computable action gaps;

sector phase transitions;

correlation lengths;

and continuum scaling laws.

If every observed configuration can be described after the fact as a compromise between coherence and obstruction, the principle would be too flexible to test.

The action therefore supplies the necessary precision.

The principle is not an independent license for interpretation.

It is a verbal summary of a specified variational model.

The finite theory can falsify particular forms of the principle.

For example, a candidate coherence action would fail if:

it possesses no stable nontrivial sectors;

its minimum depends on arbitrary gauge choice;

its defect energy diverges under refinement;

its low-energy behavior does not recover known gauge dynamics;

or its proposed sector selection is entirely parameter-tuned.

A successful action would demonstrate that a simple cost of identity transport can reproduce established gauge structure and generate additional invariant organization.

The central result of this section is therefore the transition from ontology to dynamics.

The groupoid states what local identities and equivalences are possible.

The complex states how they may be composed.

Holonomy states where identity transport fails to close trivially.

The coherence action assigns physical cost to that failure and to incompatibility with transported matter.

Boundary and topological conditions preserve nontrivial sectors.

The variational and statistical theory selects which relational configurations persist.

The next section demonstrates that, in the one-object limit and with an appropriate face cost, this construction recovers ordinary lattice gauge theory.

6. Recovery of Ordinary Lattice Gauge Theory

6.1 One-Object Reduction

The first decisive test of Univalent Relational Gauge Dynamics is whether it contains ordinary lattice gauge theory as a controlled special case.

A foundational theory cannot be considered successful merely because it introduces a more general language. It must show that established theories arise from definite restrictions, truncations, or limiting procedures. The present section demonstrates that the groupoid-valued coherence model reduces to an ordinary gauge theory when the local identity structure is restricted to one object.

Let the physical groupoid G contain a single object A.

Every morphism in G must then begin and end at A. Therefore:

Mor(G) = Aut(A).

The automorphism group of A will be denoted:

G = Aut(A).

The distinction between the groupoid and its automorphism group disappears because a one-object groupoid is equivalent to a group.

At every vertex v:

A_v = A.

Each oriented edge e carries a transport automorphism:

U_e in G.

The orientation-reversal condition becomes:

U_(e^-1) = U_e^-1.

A field configuration is therefore a conventional assignment of a group element to each oriented edge of the finite complex.

The raw configuration space is:

Conf(K,G) = G^|E_plus|,

where E_plus contains one chosen orientation for each unoriented edge.

At each vertex v, a local change of presentation is an automorphism:

g_v in G.

The transformed edge variable is:

U_e' = g_t(e) U_e g_s(e)^-1.

This is exactly the standard lattice gauge transformation law.

The local gauge group is:

G_local = product over v in V of G_v,

where each G_v is a copy of G.

Thus the general interpretation of local presentation change reduces directly to conventional local gauge freedom.

The face holonomy becomes the ordered product:

H_f = U_en ... U_e2 U_e1.

Because all morphisms belong to one group, the source-target bookkeeping of the general groupoid model is no longer required. Every edge variable can be multiplied with every compatible neighboring edge variable according to path order.

Under a local gauge transformation:

H_f' = g_vf H_f g_vf^-1,

where v_f is the chosen base vertex of the face.

The conjugacy class of H_f is gauge invariant.

For a representation rho of G, the Wilson loop:

W_gamma = Tr rho(H_gamma)

is gauge invariant for every closed loop gamma.

The matter sector also reduces to its familiar form.

Let V be a representation space of G and let:

rho : G -> GL(V).

Each vertex carries:

psi_v in V.

The edge covariant difference becomes:

D_e psi = psi_t(e) - rho(U_e) psi_s(e).

Under a gauge transformation:

psi_v' = rho(g_v) psi_v,

and:

D_e psi' = rho(g_t(e)) D_e psi.

An invariant norm produces a gauge-invariant matter action.

The one-object restriction therefore reproduces the principal ingredients of lattice gauge theory:

a fixed local gauge group;

group-valued link variables;

vertex-based gauge transformations;

plaquette holonomy;

Wilson loops;

covariantly transported matter;

and gauge-invariant actions built from class functions.

The reduction also clarifies what has been frozen.

The general groupoid theory permits:

different local identity objects;

equivalences between distinct object presentations;

several disconnected identity sectors;

different local automorphism groups;

and possible dynamical competition among those sectors.

The one-object theory suppresses all such variation.

It assumes:

one identity type;

one automorphism group;

one representation category;

and no reversible transition to another object type.

Ordinary gauge theory can therefore be interpreted as a phase or truncation in which the identity-object layer has already been selected and remains fixed.

This interpretation does not change the predictions of the recovered theory. It changes the foundational question.

Standard lattice gauge theory begins by asking:

What dynamics follow from a chosen group G?

Univalent Relational Gauge Dynamics additionally asks:

Why is the relevant local identity structure one whose automorphism group is G?

The recovery result answers the first question within the larger framework. It does not yet answer the second.

The one-object reduction should therefore be regarded as both a success and a limitation.

It is a success because the general theory contains established gauge kinematics exactly.

It is a limitation because the observed gauge group remains an input.

The later heterogeneous-groupoid analysis addresses whether the object and its automorphism group can be selected dynamically.

The reduction can be expressed as:

general groupoid model
-> restrict Ob(G) to {A}
-> identify Mor(G) with Aut(A)
-> recover G-valued lattice gauge theory.

No approximation is required at this stage. The reduction is exact.

The result is structural rather than numerical.

It shows that ordinary lattice gauge theory is embedded in the equivalence-first theory as the special case in which all local presentations belong to one fixed identity object and all transports are self-equivalences of that object.

6.2 Wilson-Type Face Action

To recover the usual lattice gauge dynamics, the face-coherence function must be chosen appropriately.

Let G be a finite or compact group and let:

rho : G -> U(N)

be a unitary representation.

For each oriented face f, define the plaquette holonomy:

H_f = U_en ... U_e1.

Choose the class function:

K(H_f) = Re Tr[I - rho(H_f)].

Equivalently:

K(H_f) = N - Re Tr rho(H_f).

The pure face action becomes:

S_face
= beta sum over f in F of w_f [N - Re Tr rho(H_f)].

When all face weights are uniform:

w_f = 1,

this is the standard Wilson plaquette form up to conventional normalization.

Another common normalization is:

S_W
= beta sum_f [1 - (1/N) Re Tr rho(H_f)].

The two expressions differ only by an overall constant factor that can be absorbed into beta.

The choice of normalization must be stated explicitly because the relationship between beta and the continuum coupling depends on it.

The Wilson-type function satisfies the required coherence conditions.

At the identity:

rho(id) = I,

so:

K(id) = 0.

Under conjugation:

H_f -> g H_f g^-1,

and therefore:

rho(H_f) -> rho(g) rho(H_f) rho(g)^-1.

The trace remains unchanged:

Tr[rho(g) rho(H_f) rho(g)^-1]
= Tr rho(H_f).

Thus K is gauge invariant.

For a unitary representation:

rho(H_f^-1) = rho(H_f)^dagger.

Therefore:

Re Tr rho(H_f^-1)
= Re Tr rho(H_f),

so the face cost is unchanged by reversing orientation.

The action is nonnegative because:

Re Tr rho(H_f) <= N

for a unitary matrix.

The identity is a minimum.

For finite groups, the Wilson-type action assigns a discrete cost to each conjugacy class.

For compact Lie groups, it provides a smooth function near the identity.

The continuum connection appears by considering weak face holonomy.

Suppose the lattice approximates a smooth manifold with lattice spacing a.

Let the edge variable be expressed schematically as:

U_e approximately equals exp(i a A_mu),

where A_mu is a Lie-algebra-valued gauge potential evaluated along the edge.

For a small elementary face in the mu-nu plane:

H_f approximately equals exp(i a^2 F_mu_nu + higher-order terms),

where F_mu_nu is the gauge curvature.

Expanding the represented holonomy:

rho(H_f)
approximately equals
I + i a^2 rho(F_mu_nu)

- (a^4 / 2) rho(F_mu_nu)^2
- higher-order terms.

For a traceless Lie-algebra representation, the linear trace term vanishes.

Then:

N - Re Tr rho(H_f)
approximately equals
(a^4 / 2) Tr[rho(F_mu_nu)^2]

- higher-order terms.

Summing over plaquettes and choosing the scaling of beta appropriately yields the continuum Yang-Mills action:

S_YM proportional to integral d^d x Tr(F_mu_nu F^mu_nu).

The precise proportionality depends on:

dimension;

lattice spacing;

representation normalization;

group convention;

and Euclidean-signature choice.

The important result is structural.

The coherence cost of failed identity closure becomes the curvature-squared action in the weak-holonomy continuum limit.

This provides a direct relation between the equivalence-first interpretation and conventional gauge dynamics:

finite loop nonclosure
-> Wilson plaquette cost
-> small-loop curvature
-> Yang-Mills action.

The continuum field strength is therefore not introduced as a separate primitive quantity.

It is recovered as the infinitesimal generator of finite identity holonomy.

This ordering is compatible with the finite constructive model.

Holonomy is exact.

Curvature is its local continuum representation.

The Wilson action also supplies an immediate computational benchmark.

For a selected finite group and finite lattice, one can compare:

the URD face action;

standard lattice-gauge calculations;

known strong-coupling expansions;

known weak-coupling behavior;

Wilson-loop expectation values;

and phase-transition structure.

If the implementation does not reproduce these results in the one-object limit, the code or formalism is incorrect.

The one-object Wilson model therefore functions as a calibration system.

The broader model should not be explored until this calibration succeeds.

The simplest benchmark cases include:

Z_2 lattice gauge theory;

Z_n Abelian lattice gauge theory;

the finite non-Abelian group S_3;

and small compact-group lattices where numerical integration is manageable.

Z_2 is especially useful because every edge variable is:

U_e = +1 or -1.

The plaquette holonomy is the product of the boundary edge values:

H_f = product over e in partial f of U_e.

The face action may be written:

S_face = beta sum_f (1 - H_f).

A face with:

H_f = +1

is flat and carries zero cost.

A face with:

H_f = -1

carries cost:

2 beta.

This model permits exact enumeration on small complexes and provides a transparent first test of:

gauge orbits;

plaquette flux;

global topological sectors;

Wilson loops;

and partition functions.

The Z_n case introduces discrete phase holonomy.

A non-Abelian finite group adds conjugacy classes and path-order dependence.

The progression from Z_2 to non-Abelian examples should be completed before moving to continuous Lie groups.

The Wilson-type action is not claimed as a novel mathematical construction.

Its role is to establish that the new theory possesses an exact established sector.

The new physical interpretation is that the action measures the cost of failed identity closure.

The scientific importance of that interpretation depends on whether it supports additional results in the more general groupoid model.

If the theory does nothing beyond reproduce Wilson's action with new terminology, it is a reformulation rather than a new physical theory.

The recovery is necessary but not sufficient.

6.3 Recovery Theorem

The central result of this section can be stated as a finite reduction theorem.

Let K be a finite oriented 2-complex.

Let G be a physical groupoid with exactly one object A.

Let:

G = Aut(A).

Assign to each oriented edge e an automorphism:

U_e in G,

subject to:

U_(e^-1) = U_e^-1.

Let the local gauge transformations be vertex assignments:

g_v in G,

acting by:

U_e -> g_t(e) U_e g_s(e)^-1.

For each face f with ordered boundary:

partial f = (e_1, ..., e_n),

define:

H_f = U_en ... U_e1.

Let rho be a unitary representation of G and define:

K(H_f)
= 1 - (1/N) Re Tr rho(H_f).

Then the Univalent Relational face action:

S_URD
= beta sum_f w_f K(H_f)

is the Wilson lattice gauge action on K for gauge group G, representation rho, and face weights w_f.

The proof follows from the preceding identifications.

Because the groupoid has one object, every transport morphism is a group element.

The general groupoid gauge transformation becomes the standard lattice link transformation.

The face holonomy becomes the ordered plaquette product.

The class function becomes the Wilson plaquette term.

The resulting sum is identical to the conventional lattice action up to normalization and geometric weighting conventions.

The theorem may be strengthened to include represented matter.

Let each vertex carry:

psi_v in V,

where V is a unitary representation space of G.

Define:

D_e psi
= psi_t(e) - rho(U_e) psi_s(e).

Let V_local(psi) be gauge invariant.

Then:

S_URD
= beta sum_f w_f K(H_f)

- kappa sum_e w_e ||D_e psi||^2
- sum_v w_v V_local(psi_v)

reduces to a standard lattice gauge theory coupled to a scalar or finite-dimensional matter field.

The exact conventional model recovered depends on:

the representation;

the local potential;

the lattice geometry;

and normalization.

This theorem establishes three levels of recovery.

First, kinematic recovery:

the variables and gauge transformations coincide with lattice gauge theory.

Second, dynamical recovery:

the face action coincides with the Wilson action.

Third, matter-coupling recovery:

the covariant edge-difference term reproduces ordinary gauge-covariant hopping or kinetic structure.

A fourth level, continuum recovery, requires additional assumptions.

One must consider a sequence of complexes K_a with lattice spacing a tending toward zero, scale the couplings appropriately, control the renormalization behavior, and show convergence of observables or effective actions.

The finite reduction theorem does not by itself prove the continuum Yang-Mills limit.

It shows that the URD model contains the same lattice regularization whose continuum behavior is studied in conventional gauge theory.

This is already important because any successful continuum limit of the recovered lattice theory is automatically available within the corresponding restricted sector of URD.

However, the broader theory may alter continuum behavior when:

object types fluctuate;

automorphism sectors compete;

defect terms are included;

higher morphisms are introduced;

or geometry becomes dynamical.

Those extensions require separate analysis.

The recovery theorem also applies to finite gauge theories.

When G is finite, the partition function is an exact finite sum:

Z
= (1 / |G|^|V|)
sum over {U_e}
exp[-S_URD({U_e}) / hbar],

subject to the treatment of boundary gauge transformations.

Equivalently:

Z
= sum over gauge orbits [U]
exp[-S_URD([U]) / hbar] / |Aut(U)|.

The equality of these forms follows from orbit-stabilizer counting.

This provides an exact test of the moduli-groupoid quantization rule.

For a small lattice, one can enumerate every edge assignment, compute every orbit, determine every stabilizer, and verify that both formulas agree.

This should be presented as a formal proposition or computational theorem in the completed paper.

The recovery theorem has a clear falsification role.

If the proposed action or transformation law fails to reduce to the Wilson theory under the one-object restriction, then the construction does not correctly generalize lattice gauge theory.

Possible failure modes include:

incorrect ordering of edge transports;

incorrect treatment of inverse orientation;

basepoint-dependent face costs;

failure of conjugacy invariance;

incorrect normalization of orbit weights;

or matter terms that are not gauge covariant.

These are not interpretive disagreements. They are mathematical errors that can be tested exactly.

The theorem therefore imposes discipline on the broader theory.

The general groupoid formulation must reproduce the established case before its new sectors are considered physically meaningful.

6.4 Interpretation of the Theorem

The recovery theorem shows that ordinary lattice gauge theory is a restricted form of equivalence-coherence dynamics.

The physical interpretation can be stated in a sequence.

A local identity object A possesses a self-equivalence group:

Aut(A) = G.

A local choice of presentation corresponds to the choice of a frame within this automorphism structure.

An edge variable U_e transports the identity frame between neighboring contexts.

A local gauge transformation changes the chosen frame at each endpoint.

A face holonomy compares identity transport around a closed elementary loop.

A Wilson plaquette term assigns cost to nontrivial return.

The resulting dynamics is ordinary lattice gauge theory.

This yields the correspondence:

gauge group = automorphisms of one fixed identity object;

gauge potential = infinitesimal description of identity transport;

link variable = finite identity transport;

field strength = infinitesimal failed identity closure;

plaquette holonomy = finite failed identity closure;

Wilson action = coherence cost of nontrivial return.

This dictionary is mathematically compatible with established gauge theory.

Its conceptual significance lies in moving gauge symmetry from the level of an externally imposed group to the self-equivalence structure of a local identity object.

The theorem does not claim that conventional physicists have been calculating the wrong action.

It claims that the same action admits a deeper relational interpretation.

Whether that interpretation is physically useful depends on whether it explains structures that ordinary gauge theory treats as inputs.

The broader groupoid model introduces precisely those additional degrees of freedom.

It may permit:

the identity object to vary;

the automorphism group to vary by sector;

equivalent object presentations to coexist;

several identity sectors to compete;

and stable defects to be interpreted as failures of global equivalence.

The one-object model can therefore be understood as a frozen-identity phase.

The object A has already been selected.

Its automorphism group G has already been selected.

Only the transport among local copies of A remains dynamical.

This suggests a hierarchy:

Level 1: fixed identity object and fixed automorphism group;

Level 2: several equivalent presentations of one identity sector;

Level 3: several competing identity sectors;

Level 4: higher morphisms among transports;

Level 5: dynamical relational geometry.

Ordinary lattice gauge theory occupies Level 1.

The proposed ToE program must explain how the higher levels reduce to that effective description under appropriate conditions.

The recovery theorem also clarifies the relation between redundancy and structure.

Gauge transformations are often described as unphysical redundancy.

That statement is correct at the level of gauge-related representatives.

However, the existence and organization of the gauge transformations are not meaningless.

They determine:

the moduli structure;

the stabilizers;

the allowed charges;

the topology of field space;

the anomaly structure;

and the possible connections.

The equivalence-first view therefore does not reify every gauge representative.

It treats the organized system of equivalences as physically foundational while continuing to identify gauge-related presentations as one physical configuration.

This avoids two opposite errors.

The first error would be to treat every gauge-related configuration as physically distinct.

The second would be to erase the transformation structure completely and retain only a bare quotient set.

The groupoid retains the equivalences without multiplying physical realities.

The theorem also provides a benchmark for the proposed interpretation of curvature.

In ordinary lattice gauge theory, plaquette holonomy is already the discrete curvature variable.

Calling it failed identity closure does not change the mathematics.

The new claim is that this interpretation can be generalized beyond a fixed group and fixed spacetime lattice.

If the same coherence principle can select object sectors, stabilize matter-like defects, and generate effective geometry, then the interpretation becomes explanatory.

If it cannot, it remains a philosophical gloss on known gauge structure.

The next sections must therefore move beyond recovery.

They must determine whether the generalized model produces:

stable defect sectors;

invariant coherence gaps;

charge representations;

dynamical symmetry selection;

or other results unavailable in the fixed-group theory.

The recovery theorem is the anchor, not the destination.

6.5 What Is Not Yet Derived

The successful reduction to lattice gauge theory must not be mistaken for a derivation of the observed laws of particle physics.

Several major inputs remain externally specified.

The gauge group G is chosen.

The representation rho is chosen.

The complex K is chosen.

The number of dimensions implicit in the lattice is chosen.

The matter fields are chosen.

The local potential is chosen.

The coupling beta is chosen.

The boundary conditions are chosen.

The continuum scaling prescription is chosen.

The one-object recovery therefore establishes compatibility with gauge theory, not the unique emergence of the Standard Model.

In particular, the theorem does not derive:

SU(3) x SU(2) x U(1);

the quotient structure by a discrete subgroup sometimes associated with the global Standard Model gauge group;

the observed hypercharge assignments;

left-right chirality;

three fermion generations;

quark and lepton representations;

the Higgs representation;

Yukawa couplings;

mixing matrices;

neutrino masses;

or the numerical values of gauge couplings.

These remain open problems.

The theorem also does not solve confinement.

For a non-Abelian gauge theory, confinement is a dynamical property diagnosed by Wilson-loop behavior, string tension, mass gaps, and the continuum limit.

Recovering the Wilson action means that the URD one-object sector inherits the confinement problem and the known lattice evidence. It does not provide a new proof.

Likewise, the theorem does not derive a mass gap in four-dimensional Yang-Mills theory.

A finite lattice has discrete excitation gaps, but demonstrating a nonzero continuum mass gap is a much stronger problem.

The proposed equivalence-inertia interpretation must not be presented as a solution unless the required scaling result is obtained.

The theorem does not derive spacetime.

The finite complex is an input relational scaffold.

Even if the lattice action approximates Yang-Mills theory on a manifold, the geometry and dimensionality have not emerged from the transport dynamics.

A later theory must show how effective distance, topology, dimension, signature, and causal structure arise.

The theorem does not derive quantum mechanics.

The Euclidean partition function supplies a statistical or path-integral-like quantization of the finite model.

It does not by itself explain:

Hilbert-space structure;

unitarity;

measurement outcomes;

the Born rule;

or the emergence of classical records.

Those require separate constructions.

The theorem does not derive gravity.

Although curvature is interpreted as failed identity closure, internal gauge curvature is not automatically spacetime curvature.

A unified model must explain how tangent, spin, internal, and possibly higher gauge structures are related without simply identifying them by analogy.

The theorem does not derive the admissibility selector.

The physical groupoid and its allowed morphisms are specified before the action is evaluated.

A deeper theory should explain why certain mathematical equivalences count as physical while others do not.

The theorem does not derive the action uniquely.

The Wilson class function is selected because it recovers established lattice gauge theory.

A true foundational derivation would explain why this form, or its continuum equivalent, follows from more primitive requirements.

Possible selection principles include:

locality;

gauge invariance;

reflection positivity;

renormalizability;

continuum scaling;

minimal derivative order;

or coherence extremization.

These constraints narrow the possibilities but may not determine a unique action.

The theorem also does not establish that the groupoid generalization is physically necessary.

A more general mathematical framework earns physical status only if its additional structure produces new explanatory or predictive results.

The decisive tests are therefore still ahead.

A first meaningful advance would be to show that a simple heterogeneous groupoid action dynamically selects one automorphism sector over others.

A second would be to produce a stable defect whose charge and energy arise from the relational structure rather than from inserted labels.

A third would be to derive a robust relation among defect energy, automorphism representation, and topology.

A fourth would be to show that the generalized theory possesses a controlled continuum limit distinct from, but containing, ordinary gauge theory.

A fifth would be to identify an observable deviation or selection rule that could in principle be tested.

The limitations of the recovery theorem should be stated explicitly because conceptual unification can otherwise be mistaken for physical derivation.

The theorem proves:

ordinary lattice gauge theory is contained in the one-object sector of Univalent Relational Gauge Dynamics.

It does not prove:

the universe must select that sector;

the observed gauge group is inevitable;

matter is completely explained;

or the full theory is a Theory of Everything.

The result nevertheless marks an essential threshold.

The equivalence-first framework is no longer disconnected from established gauge dynamics.

Its core action possesses a precise conventional limit.

Its language of automorphisms, transport, holonomy, and coherence is not merely metaphorical.

The next task is to determine whether the additional groupoid structure generates matter-like defects, charge sectors, and invariant mass costs rather than simply reproducing a known fixed-group model.

7. Defect Matter, Charge, and Mass

7.1 Defects as Obstruction Classes

The recovery of ordinary lattice gauge theory establishes that the one-object sector of Univalent Relational Gauge Dynamics reproduces familiar gauge transport and curvature. The next question is whether the broader theory can generate matter-like structures rather than merely describe gauge fields on a fixed complex.

The proposed answer is that particle-like matter may arise as a stable obstruction to the global trivialization of identity transport.

A defect is not defined simply as a region of high action or nonzero curvature. Local curvature fluctuations may appear and disappear without carrying a persistent physical identity. A defect must instead possess an invariant obstruction that prevents it from being removed by admissible local transformations while remaining within the same physical sector and boundary conditions.

Let Phi denote a configuration of identity objects, edge transports, and optional matter variables:

Phi = ({A_v},{U_e},{psi_v}).

A trivial configuration is one that can be transformed, through admissible gauge transformations and continuous or discrete deformations allowed by the theory, into a chosen vacuum configuration Phi_0.

A defect sector consists of configurations that cannot be so transformed without violating at least one of the following:

the admissibility rules;

the boundary conditions;

the global topology;

the identity-sector assignment;

a conserved obstruction invariant;

or a finite-action continuity condition.

The defect is therefore defined relationally.

It is not merely an object placed into the field. It is the failure of the field's equivalence structure to become globally trivial.

The general obstruction problem can be stated as follows.

Given a configuration Phi, determine whether there exists a family of local gauge transformations g_v and admissible deformations Phi(t), with:

Phi(0) = Phi,

Phi(1) = Phi_0,

such that every intermediate configuration remains allowed and satisfies the same physical boundary data.

If such a deformation exists, Phi belongs to the vacuum-connected sector.

If no such deformation exists, Phi lies in a nontrivial defect sector.

The set of defect sectors may be denoted:

D(K,G) = pi_0(Conf_phys),

where pi_0 refers to connected components of the physical configuration space under the chosen class of admissible deformations.

For finite configuration spaces, connectedness must be defined through a specified set of allowed local update moves.

For continuous groups and fields, it may be defined through continuous paths in configuration space.

The defect classification depends on the physical model. Different theories support different obstruction invariants.

Possible defect labels include:

conjugacy classes of holonomy;

winding numbers;

homotopy classes;

cohomology classes;

flux values;

domain-wall classes;

vortex numbers;

monopole charges;

bundle obstruction classes;

representation-theoretic labels;

or higher-categorical cocycles.

The finite model should begin with the simplest cases in which the obstruction can be computed exactly.

A first example is global holonomy on a complex with a noncontractible cycle.

Suppose every elementary face satisfies:

H_f = id,

but a loop gamma winding around a noncontractible direction has:

H_gamma != id.

The configuration is locally flat but globally nontrivial.

If the holonomy cannot be removed by a gauge transformation compatible with the topology and boundary conditions, its conjugacy class labels a global sector.

For an Abelian group, the holonomy itself may be the sector label.

For a non-Abelian group, the gauge-invariant label is generally its conjugacy class or a related class function.

A second example is fixed boundary flux.

Suppose the boundary conditions require a nontrivial total holonomy around the exterior boundary.

The interior field cannot be globally gauge equivalent to trivial transport.

The imposed flux may distribute across the complex or localize into a smaller region.

The minimum-action configuration within that boundary sector defines the preferred defect profile.

A third example arises from matter winding.

Let the matter field psi take values in a vacuum manifold M_vac determined by the minima of its local potential.

If the boundary values define a nontrivial map:

boundary K -> M_vac,

then the field may be unable to extend smoothly or coherently through the interior while remaining everywhere in the vacuum manifold.

A defect core must appear where:

psi leaves M_vac;

psi vanishes;

the identity object changes;

or transport becomes nontrivial.

The topological class of the boundary map labels the defect.

A fourth example is a domain wall between distinct identity sectors.

Suppose one region is dominated by an object type A and another by an object type B, where A and B lie in different physical groupoid components.

No reversible admissible morphism connects them.

The interface is then a genuine identity discontinuity rather than a gauge choice.

Its energy arises from the incompatibility or transition cost between sectors.

The wall may be stable if the two phases are degenerate or if boundary conditions require both.

A fifth example is nontrivial higher coherence.

In a higher-groupoid extension, edge transports may be locally trivial while face or higher-cell data carry nontrivial cocycles.

The defect then resides not in ordinary holonomy but in failure of higher trivialization.

Such defects may correspond to higher-form charges or extended objects rather than point-like particles.

The general defect classification should distinguish several levels.

A local curvature excitation is a region where some face holonomies differ from the identity.

A topological sector is a class of configurations distinguished by a global obstruction invariant.

A stable defect is a localized or extended minimum of the action within a nontrivial sector.

A particle-like excitation is a stable defect whose collective behavior under translation, interaction, and propagation can be approximated by an effective localized degree of freedom.

These terms should not be used interchangeably.

Every particle-like defect belongs to a nontrivial or metastable sector.

Not every nontrivial sector produces a localized particle.

Some sectors may correspond to uniform flux, global topology, domain structure, or extended defects.

The finite model must determine which type occurs.

A useful formal definition is the following.

A defect class D is a subset of physical configurations satisfying:

D(Phi) = d,

where d is a gauge-invariant obstruction label.

The vacuum class satisfies:

D(Phi_0) = 0.

The minimum action in sector d is:

S_min(d)
= inf over Phi with D(Phi)=d of S_URD(Phi).

The excess coherence cost is:

Delta S(d)
= S_min(d) - S_min(0).

A stable defect requires:

Delta S(d) > 0;

the infimum is attained or approached by a localized configuration;

small admissible perturbations do not remove the sector label;

and the minimizing configuration does not dissolve as system size increases under fixed physical conditions.

Localization must be defined quantitatively.

Let epsilon_f denote the local face action density:

epsilon_f = beta w_f K(H_f).

Let epsilon_e denote the matter-transport density:

epsilon_e = kappa w_e ||D_e psi||^2.

Let epsilon_v include potential and identity-sector terms.

A defect is localized when the excess action density relative to the vacuum is concentrated within a bounded region whose characteristic size remains finite or scales in a controlled physical manner as the total complex grows.

One may define a defect center by the weighted centroid of excess action.

A defect radius may be defined from the second moment of that distribution.

A localization ratio may compare the excess action inside a selected radius with the total excess action.

These definitions permit finite-size scaling.

A defect that spreads uniformly over the whole lattice as the lattice grows is not a localized particle-like object, although it may still represent a global sector.

The obstruction label must be independent of gauge choice.

For a holonomy defect, its conjugacy class is gauge invariant.

For a winding defect, the winding number must be unchanged under admissible transformations.

For a domain wall, the identity-sector components on each side must be physical rather than arbitrary labels.

For higher defects, the cocycle class must descend to the moduli structure.

The defect must also be independent of arbitrary subdivision.

Refining the complex may change the number of faces carrying curvature but should not change the total topological class.

A physically meaningful defect label must possess a refinement map or continuum interpretation.

This condition is essential for distinguishing genuine obstruction from lattice artifact.

The defect concept provides the first strong test of the equivalence-first ontology.

The framework claims that persistent physical objects may arise from coherence obstruction.

A finite model can test this directly.

If every nontrivial local structure can be gauge transformed away or continuously relaxed into the vacuum, then the model does not generate matter.

If nontrivial sectors exist but their minimum configurations are delocalized or unstable, the model generates topology but not particle-like matter.

If localized stable sectors appear with invariant charges and finite energy, the model begins to support the proposed interpretation.

The transition from field to matter is therefore:

identity transport defines local coherence;

nontrivial topology or boundary data prevents complete closure;

the action minimizes the unavoidable obstruction;

the minimum localizes;

the localized obstruction persists;

and the persistent pattern behaves as an object.

Matter is then not inserted as a primitive substance.

It is the stable form taken by a relational inconsistency that cannot be eliminated without changing the physical sector.

The word inconsistency must again be interpreted carefully.

The configuration is mathematically consistent.

The obstruction is a failure of global trivialization, not a contradiction.

The defect is a coherent nontriviality.

This formulation is analogous to a knot.

A knot is not an error in the string. It is a stable global arrangement that cannot be removed without cutting the string or passing through a forbidden configuration.

Likewise, a relational defect is a stable organization of equivalence structure that cannot be removed through admissible local transformations.

The finite theory should classify defects in increasing order of complexity.

The first class should use Abelian holonomy, where obstruction labels are easiest to compute.

The second should use a finite non-Abelian group, where conjugacy and path ordering matter.

The third should use matter winding.

The fourth should use heterogeneous identity sectors.

The fifth should introduce higher coherence defects.

At each stage, the theory should determine:

the obstruction label;

the minimum-action configuration;

the degeneracy of the sector;

the localization profile;

the response to perturbation;

and the behavior under lattice refinement.

This program turns the phrase particles as defects into a sequence of exact mathematical and computational questions.

7.2 Charge as Automorphism Response

A stable defect becomes physically richer when it transforms nontrivially under the automorphism structure of the surrounding identity sector.

The theory proposes that charge is not an arbitrary label attached to a particle-like object. It is the way that object responds to admissible local self-equivalences.

Let A be the local vacuum identity object.

Its automorphism group is:

G = Aut(A).

Suppose a stable defect sector D supports a state space V_D.

The action of local automorphisms on that defect is described by a representation:

rho_D : G -> GL(V_D).

The representation class of rho_D is the charge type of the defect.

This generalizes the conventional relationship between charge and gauge-group representation.

In ordinary gauge theory, matter fields are assigned representations of a chosen gauge group.

In the equivalence-first framework, the gauge group is interpreted as the automorphism group of local physical identity, while the charge describes how a persistent coherence pattern transforms under those self-equivalences.

The conceptual order becomes:

local identity structure A;

self-equivalence group Aut(A);

stable defect sector D;

representation rho_D of Aut(A);

physical charge associated with rho_D.

The representation must be compatible with gauge equivalence.

If the local identity object is replaced by an equivalent presentation B through:

u : A -> B,

then the automorphism group is transported by conjugation:

g in Aut(A)
maps to
u g u^-1 in Aut(B).

The defect representation must transform correspondingly.

The physical charge is therefore not the literal matrix form of rho_D in one chosen presentation.

It is the equivalence class of the representation under admissible changes of identity frame.

For an Abelian group, irreducible representations are one-dimensional in the simplest compact cases, and the charge may be represented by a phase weight.

For U(1):

rho_q(exp(i theta))
= exp(i q theta).

The parameter q labels the charge.

For a finite cyclic group Z_n, the irreducible charge sectors are discrete characters.

For a non-Abelian group, charge corresponds to an irreducible or composite representation and may carry multiple internal components.

The automorphism-response interpretation should not be presented as a wholly new mathematical fact.

Gauge charges are already associated with representations of gauge groups.

The distinctive proposal is that both the gauge group and its charged sectors may arise from a deeper identity-and-defect structure rather than being independently postulated.

A defect may carry charge in several related ways.

First, its internal state space may transform under rho_D.

Second, the defect may impose nontrivial holonomy detectable by probes.

Third, it may act as a source for gauge flux.

Fourth, it may be labeled by a topological class related to the automorphism group.

Fifth, its fusion with another defect may follow representation-theoretic composition rules.

These notions should be distinguished.

Representation charge describes transformation behavior.

Flux charge describes the gauge field sourced or enclosed by the defect.

Topological charge describes an obstruction class.

In some theories they are related; in others they are distinct.

The finite model should not assume their equivalence without proof.

A useful operational definition of charge uses transport around the defect.

Let gamma be a loop enclosing a localized defect.

The holonomy:

H_gamma

acts on a probe representation V_probe through:

rho_probe(H_gamma).

If this action differs from the vacuum value and remains invariant under deformations of gamma that do not cross the defect, the defect carries a measurable gauge response.

The corresponding conjugacy or character data can serve as a charge observable.

For an Abelian theory, the enclosed holonomy may directly determine a flux or phase.

For a non-Abelian theory, the conjugacy class and representation characters are more appropriate.

Another definition uses the residual action of global or boundary gauge transformations.

Gauge transformations that vanish at the boundary are pure redundancy.

Transformations with nontrivial boundary value may act on physical states.

A defect state may transform under this boundary symmetry.

The corresponding representation labels its conserved charge.

This distinction between interior gauge redundancy and boundary physical symmetry is important.

The same automorphism structure can act as redundancy locally and as a physical symmetry on boundary or asymptotic data.

The theory must specify which transformations are divided out and which remain as observable actions.

Charge conservation should follow from symmetry of the action.

If the coherence action is invariant under a continuous family of global automorphisms, a Noether-type conserved quantity may exist.

In a finite or discrete theory, conservation may instead appear as:

sector preservation;

representation fusion rules;

Gauss-law constraints;

or invariance of total charge under local updates.

The finite model should define a discrete Gauss law.

For a vertex v, the outgoing and incoming transport and matter representation data should satisfy a gauge constraint.

In Hamiltonian lattice gauge theory, the generator of local gauge transformation imposes such a condition.

In the current Euclidean formulation, an equivalent constraint may emerge from integration over vertex gauge transformations or from the structure of admissible states.

A charged defect may then be identified by a nontrivial source term in this local constraint.

The total charge of a closed system should be constrained by the global boundary conditions and topology.

The representation structure also determines composition.

Suppose two defects carry representations:

rho_1 and rho_2.

A composite system transforms in:

rho_1 tensor rho_2.

The possible resulting charge sectors are determined by the decomposition of this tensor product into irreducible components.

Thus defect fusion may follow:

D_1 x D_2 -> direct sum over c of N_(12)^c D_c,

where N_(12)^c records the multiplicity of charge sector c.

In an Abelian theory, charges add according to the group of characters.

In a non-Abelian theory, fusion can produce several possible composite sectors.

This provides a route from automorphism structure to particle-combination rules.

Antiparticles may be represented by dual or conjugate charge sectors.

If D carries rho_D, its antiparticle candidate may carry:

rho_D^*,

the dual representation.

The tensor product:

rho_D tensor rho_D^*

contains the trivial representation under common conditions.

This permits particle-antiparticle annihilation into a neutral sector.

However, representation duality alone does not guarantee physical antiparticles.

The theory must also provide:

opposite topological or flux charge;

compatible energy;

a dynamical creation and annihilation process;

and appropriate statistics.

These are later requirements.

Charge quantization may arise naturally when the allowed irreducible representations form a discrete set.

For finite groups, charge sectors are automatically discrete.

For compact Lie groups, representation weights are quantized by the group structure.

The deeper question is whether the relevant automorphism group itself is dynamically selected.

If it is, then the allowed charges follow from the representation theory of the selected identity object.

This is a promising route toward deriving charge spectra.

Yet several limitations remain.

The current model does not derive the Standard Model representation content.

It does not explain chirality.

It does not explain why observed matter occupies particular irreducible representations.

It does not derive hypercharge normalization.

It does not explain anomaly cancellation.

It does not derive three generations.

These problems should be stated explicitly.

The immediate constructive goal is narrower:

show that a stable defect sector carries a well-defined, gauge-invariant automorphism representation;

show that this representation can be measured through loop or boundary observables;

show that composite defects obey the corresponding representation rules;

and show that the charge label remains stable under admissible local dynamics.

A finite test may proceed as follows.

Choose a small group G.

Construct a complex supporting localized defect sectors.

Determine the stabilizer and transformation action on the defect-state space.

Compute the representation characters.

Apply random local gauge transformations and verify that the representation class remains unchanged.

Combine two defects and calculate the decomposition of their joint state space.

Test whether local dynamics preserves total charge.

If these conditions hold, the theory has produced a concrete realization of charge as automorphism response.

The result would still reproduce a familiar gauge-theoretic principle, but within a broader constructive setting in which the charged object is itself an emergent coherence defect.

The physical dictionary would then be:

identity object -> local gauge symmetry;

automorphism group -> gauge group;

defect state space -> matter multiplet;

representation -> charge type;

conjugate representation -> antiparticle charge;

tensor product -> composite charge;

invariant subspace -> neutral combination.

This is the first step toward deriving matter classification from relational structure.

7.3 Mass as Invariant Coherence Cost

The most distinctive physical proposal of the theory is that mass measures the minimum invariant cost required to maintain and transport a persistent identity pattern.

This proposal must be formulated with great care.

A coefficient appearing in a local potential is not automatically a derived mass.

A finite action difference is not automatically a relativistic rest energy.

A localized high-action region is not automatically a particle.

The theory must distinguish several quantities:

local action density;

minimum defect action;

static defect energy;

propagation energy;

correlation mass;

inertial response;

and relativistic rest mass.

The phrase mass as equivalence inertia is justified only if these quantities are connected through a controlled dynamical construction.

Begin with a defect sector d.

Let:

C_d

denote the set of physical configurations carrying defect label d and satisfying specified boundary conditions.

Let:

C_0

denote the vacuum sector.

The minimum Euclidean action values are:

S_min(d)
= inf over Phi in C_d of S_URD(Phi),

S_min(0)
= inf over Phi in C_0 of S_URD(Phi).

Define the defect action gap:

Delta S_d
= S_min(d) - S_min(0).

This quantity is gauge invariant if:

the action descends to physical equivalence classes;

the sector label is gauge invariant;

and the minimization is performed over complete physical sectors rather than arbitrary representatives.

Delta S_d is the least excess coherence action required to realize the defect under the chosen finite conditions.

It is the first candidate measure of equivalence inertia.

However, Delta S_d should initially be called a coherence gap, not a mass.

To obtain a physical mass, the model requires a time or Euclidean-time direction.

Let the complex contain a distinguished sequence of spatial slices indexed by Euclidean time t.

A static defect configuration repeated across time defines a worldline-like extended pattern.

The Euclidean action for a persistent defect over duration T may scale as:

S_d(T) - S_0(T)
approximately equals
E_d T

for sufficiently large T.

Then:

E_d
= limit as T -> infinity of
[S_d(T) - S_0(T)] / T

defines the static defect energy.

In units with c explicit, the rest mass candidate is:

m_d = E_d / c^2.

This definition is closer to standard field theory.

It measures the action cost per unit Euclidean time of maintaining the defect's identity.

The phrase equivalence inertia then has a precise meaning:

the defect persists because it occupies a stable obstruction class;

maintaining that obstruction through time requires an irreducible action cost;

that cost appears as rest energy.

A second and often more robust definition uses correlation functions.

Let O_d be a gauge-invariant operator that creates or detects the defect sector.

Consider the Euclidean two-point function:

C_d(t)
= <O_d(t) O_d(0)>.

At large t, if a lowest excitation of mass m_d dominates:

C_d(t)
approximately equals
A exp(-m_d c^2 t / hbar),

or in lattice units:

C_d(t)
approximately equals
A exp(-m_d t).

The exponential decay rate defines the mass gap.

This procedure avoids equating a raw action minimum directly with mass.

It extracts the energy of a propagating excitation from observable correlations.

A transfer-matrix formulation provides another route.

If the Euclidean theory defines a positive transfer matrix T, its eigenvalues may be written:

lambda_n = exp(-a_t E_n / hbar),

where a_t is the temporal lattice spacing.

The energy spectrum is:

E_n = -(hbar / a_t) log lambda_n.

The lowest nonvacuum state in defect sector d has energy E_d.

The corresponding rest mass is E_d / c^2 when a relativistic low-energy dispersion relation is established.

The equivalence-first interpretation remains:

the eigenstate corresponds to a persistent coherent obstruction;

its energy measures the cost of carrying that identity pattern through time.

A complete theory should also examine motion.

Let a defect be displaced across spatial slices.

A worldline gamma records the sequence of defect positions.

The effective action may take the low-velocity form:

S_eff
approximately equals
integral dt [m_d c^2 + (1/2) m_inertial v^2 + higher terms].

If the coefficient of the kinetic term equals the rest-energy parameter divided by c^2, the defect exhibits conventional inertial behavior.

In a relativistic regime, the effective worldline action should approach:

S_eff = -m_d c integral ds.

This is a demanding recovery condition.

The finite theory should not assume it at the outset.

It should test whether localized defect propagation produces an approximately relativistic dispersion relation:

E(p)^2
approximately equals
m_d^2 c^4 + p^2 c^2

at low momentum and large scale.

Only then can the coherence gap be identified with physical inertial mass in the conventional sense.

The invariant aspect of the definition is essential.

A raw defect profile may be represented in many gauges, on different but equivalent subdivisions, or with different local object labels.

The physical mass must remain unchanged under all such presentation changes.

The minimization over equivalent presentations can be expressed as:

M_inv(d)
= inf over Phi' equivalent to Phi and in sector d of
E(Phi').

This removes artificial cost caused by a poor choice of representative.

For example, a gauge-dependent field configuration may appear highly variable even though a gauge transformation makes it simple.

The energy should not count this descriptive complexity as physical mass.

The minimization must also include local deformations within the same sector.

A defect's mass is not the cost of an arbitrary representative.

It is the least energy compatible with carrying that invariant obstruction.

This is analogous to finding the ground-state energy within a topological sector.

The mass may contain several contributions:

face-curvature energy;

matter-gradient energy;

local potential energy;

identity-sector maintenance cost;

interface energy;

and quantum fluctuation corrections.

The theory should decompose these contributions but treat only the total invariant result as physical.

The proposed definition can be written:

$$m_d c^2 = E_{min}(d) - E_{min}(0),$$

where $E_{min}(d)$ is the lowest energy in defect sector d.

At the quantum level:

$m_d c^2$

is the difference between the lowest Hamiltonian eigenvalue in the defect sector and the vacuum energy, subject to finite-volume and renormalization conventions.

This is a standard spectral structure with a new ontological interpretation.

The novelty is not the equation $E = mc^2$ or the use of energy gaps.

The novelty is the claim that the sector exists because of an obstruction in admissible equivalence and that its irreducible energy is the cost of preserving that obstruction as a physical identity.

The mass definition must be tested against several possible failures.

First, gauge dependence.

If different gauge representatives produce different extracted masses, the definition is invalid.

Second, lattice dependence.

If the mass changes arbitrarily under refinement without a scaling law, it is a discretization artifact.

Third, boundary dependence.

Finite-volume boundaries may affect the defect energy. The infinite-volume or controlled finite-size limit must be studied.

Fourth, instability.

If the sector decays into lower-energy configurations with the same total charge, the original defect is not a stable particle.

It may represent a resonance.

Fifth, delocalization.

If the excess energy spreads across the entire system, the excitation may not be particle-like.

Sixth, parameter insertion.

If the mass value is fixed directly by a local coefficient chosen to equal the desired result, it has not been derived.

Seventh, absence of relativistic propagation.

A coherence gap may exist without behaving as relativistic mass.

The framework would then describe a condensed-matter-like excitation rather than a fundamental particle.

These distinctions increase rather than reduce the value of the proposal.

They transform mass as equivalence inertia from a metaphor into a falsifiable program.

A minimal computational demonstration should calculate:

the vacuum minimum;

the minimum in one defect sector;

the action or energy gap;

the defect profile;

the two-point correlation decay;

the dependence on lattice size;

the dependence on lattice spacing;

and the dispersion relation if motion is included.

The strongest result would be a mass value determined by topology, automorphism structure, and a small number of universal couplings rather than by an independent mass parameter.

A still stronger result would be a ratio of defect masses that is robust across scale and parameter variation.

Mass ratios are often more meaningful than absolute masses because overall units may depend on the lattice scale.

If two defect classes d_1 and d_2 produce:

m_(d_1) / m_(d_2)

as a stable dimensionless number, the theory has generated a genuine quantitative relation.

The long-term goal is to derive observed particle masses or coupling-dependent mass patterns.

The immediate goal is to demonstrate that a gauge-invariant coherence defect possesses a calculable, presentation-independent energy gap.

That is the first necessary step.

7.4 Stability

Persistence is the defining characteristic of a particle-like defect.

A localized configuration that disappears under an arbitrarily small perturbation does not possess a robust physical identity.

The theory must therefore define stability at several levels.

Topological stability occurs when the defect label cannot change through any admissible local deformation that preserves the boundary conditions.

Energetic stability occurs when the defect configuration is a local or global minimum of the energy within its sector.

Dynamical stability occurs when time evolution does not cause the defect to decay or disperse.

Quantum stability occurs when no lower-energy final state with the same conserved quantum numbers is available.

Metastability occurs when decay is possible but suppressed by an action barrier.

These forms of stability should be analyzed separately.

A topologically stable defect belongs to a sector disconnected from the vacuum.

No continuous admissible path in physical configuration space connects it to the trivial sector.

Its removal requires:

changing boundary conditions;

crossing a forbidden configuration;

allowing a singularity;

changing the identity groupoid;

or creating an oppositely charged defect.

Topological stability is robust because it does not depend on the detailed shape of the action.

The action determines the defect profile and energy, but the obstruction prevents complete removal.

Energetic stability is more model dependent.

Within a fixed defect sector, the minimizing configuration must resist small perturbations.

Let Phi_d be a stationary defect solution.

For continuous fields, the second variation of the energy should be nonnegative on physical perturbations after gauge zero modes are removed.

For finite discrete models, every allowed local update near Phi_d should increase or preserve the action.

Gauge directions must not be mistaken for instabilities.

A local gauge transformation changes the representative but not the physical configuration.

The Hessian or update analysis should therefore be performed on gauge-fixed variables or on the quotient space.

Dynamical stability requires an explicit update law or Hamiltonian evolution.

A Euclidean action alone defines statistical weights but not real-time motion.

The theory must eventually construct:

a transfer matrix;

a Hamiltonian;

a discrete-time evolution rule;

or a Lorentzian action.

A defect is dynamically stable when its identity label and localized profile persist under that evolution, apart from translation, internal rotation, or other symmetry motion.

A defect may radiate excess energy and relax to the minimum configuration in its sector while remaining the same physical object.

Such relaxation is compatible with persistence.

The invariant identity lies in the sector and charge, not in every microscopic detail.

This illustrates the equivalence-first concept of objecthood.

A particle-like defect is not a frozen field arrangement.

It is a family of configurations connected by admissible deformation and transport that preserve the relevant obstruction and representation data.

Its identity type includes:

its defect class;

its charge representation;

its minimum-energy sector;

its transformation behavior;

and its allowed deformations.

Quantum stability requires conservation laws.

A defect with charge q cannot decay into neutral states if total charge is conserved.

It may decay into several defects whose combined charge equals q if energetically allowed.

The representation-fusion rules determine possible channels.

For defects D_a, D_b, and D_c, a decay:

D_a -> D_b + D_c

is allowed only if the charge representation of D_a appears in the tensor product:

rho_b tensor rho_c,

and if all other conserved obstruction labels match.

Energy conservation additionally requires:

m_a >= m_b + m_c

in the rest frame, with appropriate binding and kinetic terms.

A stable particle corresponds to the lowest-energy state carrying its complete set of conserved labels.

A higher-energy state in the same sector may be an excitation or resonance.

This framework naturally separates identity from state.

Several states may share one defect identity while differing in:

position;

momentum;

internal excitation;

gauge presentation;

or surrounding field configuration.

The persistent object is the stable relational sector, not one microscopic state.

Stability should be tested computationally.

Begin with a minimized defect configuration.

Apply perturbations of increasing strength:

random edge changes;

matter-field noise;

local object substitutions;

geometric refinement;

boundary perturbations;

or nearby defect insertion.

Then relax or evolve the configuration.

Measure:

whether the defect label survives;

whether the energy returns near its minimum;

whether the defect remains localized;

whether it moves;

whether it splits;

and whether it annihilates with an opposite defect.

A stability probability can be estimated across random perturbations.

A critical perturbation threshold may separate reversible deformation from sector change.

The lifetime of a metastable defect may be measured under stochastic or quantum dynamics.

The dependence of lifetime on action barrier should be tested.

For thermally activated decay:

tau

may scale approximately as:

tau approximately equals tau_0 exp(Delta E_barrier / k_B T).

For quantum tunneling, the decay rate may scale with a Euclidean instanton action.

These mechanisms provide concrete ways for relational identities to be stable but not eternal.

Defect interaction is another stability test.

Two like-charged defects may repel, attract, merge, or form a bound state.

A defect and its inverse may annihilate into the vacuum sector.

The total obstruction class must be preserved.

For an Abelian additive charge:

q_total = q_1 + q_2.

For non-Abelian or categorical charges, composition may follow more complex fusion rules.

A bound state is stable when its total energy is lower than the energy of separated constituents with the same total charge.

The binding energy is:

$$E_{bind} = E_{bound} - (E_1 + E_2).$$

A negative value indicates energetic binding under the chosen convention.

Within the equivalence-first interpretation, binding reduces the total cost of sustaining the combined coherence pattern.

This provides a possible route to interpreting binding energy as a reorganization of equivalence maintenance.

The model should also test translational stability.

On a regular homogeneous complex, defect positions related by lattice translation should have equal energy away from boundaries.

A localized defect may then possess a translational zero mode or a small lattice pinning potential.

As the lattice is refined, artificial pinning should decrease if a continuum limit exists.

If the defect can be moved through a sequence of low-action configurations, its identity is transportable.

This transportability is essential for particle interpretation.

A structure that can exist only at one fixed lattice site is more likely a defect of the discretization than a physical excitation.

The defect must retain its charge and internal identity while moving.

The transport maps between successive defect positions constitute a higher-level identity path in configuration space.

Thus the particle's worldline is not merely the path of a point.

It is the coherent transport of a defect class through a sequence of local presentations.

This is one of the clearest applications of Univalent Relational Dynamics.

Object persistence through time is represented by a family of equivalences between neighboring defect configurations.

The particle exists because those configurations can be coherently identified.

Stability is therefore not absolute immobility.

It is persistence of invariant structure across deformation and motion.

7.5 Mass Gap

A theory may possess nontrivial defect sectors but still fail to generate a meaningful particle scale.

The mass gap measures the energy separation between the vacuum and the lowest nontrivial excitation.

Let E_0 be the vacuum energy.

Let E_1 be the lowest energy of any physical excitation not gauge equivalent to the vacuum.

The mass gap is:

Delta
= E_1 - E_0.

If the lowest excitation is a localized defect with relativistic behavior, then:

Delta = m_gap c^2.

In a finite system, the energy spectrum is discrete under broad conditions, so a nonzero finite-size gap is common.

The significant question is whether the gap remains nonzero in the relevant infinite-volume and continuum limits.

Let L denote the linear system size and a the lattice spacing.

The finite gap is:

Delta(L,a).

A physically meaningful continuum mass gap requires a scaling procedure in which:

L -> infinity;

a -> 0;

physical volume and renormalized couplings are controlled;

and:

Delta_phys
= limit of Delta(L,a) / appropriate time or length units

exists and is positive.

The finite model cannot claim a continuum mass gap merely because a small lattice has separated energy levels.

Finite-size scaling is essential.

Several scenarios are possible.

The gap may approach a positive constant.

This indicates a massive phase.

The gap may vanish as a power of system size.

This may indicate a massless or critical phase.

The gap may vanish exponentially.

This may indicate tunneling between nearly degenerate sectors.

The gap may diverge in lattice units but remain finite after rescaling.

The gap may fail to converge because no continuum limit exists.

These possibilities should be analyzed through numerical scaling.

Correlation functions provide the practical diagnostic.

For a gauge-invariant operator O carrying the quantum numbers of the lowest excitation:

C(t)
= <O(t)O(0)> - ^2.

At large t:

C(t)
approximately equals
A exp(-Delta t / hbar).

On a periodic Euclidean lattice, forward and backward propagation produce a cosh-like form.

The effective mass estimator may be defined from ratios of neighboring correlation values.

A plateau in the effective mass over a range of t supports extraction of the gap.

Multiple operators should be used to reduce contamination from higher excitations.

A variational basis of defect-creating operators may improve the estimate.

The mass-gap calculation should compare several sectors.

Vacuum fluctuations may produce the lowest excitation without topological charge.

A charged defect may be heavier.

A neutral bound state may be lighter than a pair of charged defects.

The spectrum should therefore be organized by:

defect class;

charge representation;

momentum;

parity or orientation;

and other discrete symmetries.

In a finite groupoid theory, the automorphism representation provides a natural sector decomposition.

The transfer matrix may be block diagonal in charge sectors.

The lowest eigenvalue in each block defines the lightest state carrying that charge.

This gives:

Delta_r
= E_min(r) - E_0,

where r labels an irreducible automorphism representation.

The set {Delta_r} is the charge-resolved mass spectrum.

A strong result would be that these gaps are determined by the coherence action and defect topology with no independent mass parameters.

An even stronger result would be that the ratios:

Delta_r / Delta_s

approach universal values under refinement.

The mass gap also measures correlation length.

In Euclidean units:

xi approximately equals hbar c / Delta.

A nonzero gap corresponds to finite correlation length.

A vanishing gap corresponds to long-range or critical behavior.

This relation connects defect mass to the spatial scale over which coherence disturbances propagate.

The equivalence-first interpretation is:

a massive excitation is a coherence obstruction whose influence decays over a finite relational distance;

a massless excitation supports long-range coherent propagation without a finite rest-energy gap.

This interpretation should be treated as a consequence of the spectral theory, not as a substitute for it.

The model should also distinguish confinement-induced gaps from topological defect gaps.

In a non-Abelian gauge theory, physical excitations may be gauge-invariant composites even when the underlying matter fields are not observable in isolation.

The lightest gauge-invariant state may define the mass gap.

A defect interpretation may classify such states through flux tubes, loop excitations, or bound coherence structures.

However, recovering a finite-lattice mass spectrum does not solve the general continuum confinement problem.

The paper should remain precise about the scope of its result.

A first computational mass-gap demonstration can use a small finite group.

For Z_2 gauge theory, one can calculate the transfer matrix or partition function exactly on a narrow lattice.

Excitation sectors can be labeled by flux or Wilson-loop eigenvalues.

The finite-size gap can be compared with known results.

This calibrates the implementation.

The new groupoid model can then introduce:

multiple identity objects;

sector transitions;

or localized defects.

The question becomes whether new gaps appear that are absent from the fixed-group theory.

A non-inserted gap would be especially important.

Suppose all local identity objects have zero baseline cost and the action contains only universal coherence terms.

If the dynamics nevertheless generates a stable positive defect gap, the model has produced an emergent mass scale.

The absolute value still depends on the overall coupling and lattice unit, but the existence and ratios may be nontrivial.

Dimensional transmutation may also occur in continuum-like non-Abelian models.

A dimensionless coupling can generate a physical scale through renormalization.

The equivalence-first framework might reinterpret that scale as the cost of sustaining nontrivial coherence sectors.

But such a connection must be derived, not asserted.

The mass-gap program therefore has three levels.

Level 1:

compute exact finite gaps and verify gauge invariance.

Level 2:

show that defect-associated gaps are stable under increasing system size.

Level 3:

establish a continuum scaling limit with a positive physical gap.

The first level is achievable within the present constructive paper.

The second may require extensive computation.

The third is a major field-theoretic challenge.

The paper should claim only the level actually demonstrated.

A successful finite result would still be important.

It would show that the equivalence-first dynamics generates a spectrum of persistent coherent obstructions with calculable energy costs.

That is the first operational realization of mass as equivalence inertia.

7.6 Failure Conditions and Scientific Tests

The defect interpretation must be exposed to explicit failure conditions.

Without them, any localized structure could be redescribed after the fact as matter, and any action cost could be called mass.

The theory therefore requires strict rejection criteria.

The first failure condition is absence of gauge-invariant defect classification.

If the proposed defect label changes under an admissible local gauge transformation, it is a presentation artifact.

It cannot represent physical matter.

The second failure condition is removability.

If every candidate defect can be eliminated by an admissible local deformation while preserving the boundary conditions and physical sector, it is not topologically or structurally protected.

It may be a transient fluctuation but not a stable identity.

The third failure condition is lack of localization.

If the minimum nontrivial sector spreads uniformly across the full system and its characteristic size grows with the total lattice size, it does not behave as a localized particle.

It may still represent global flux or a different vacuum sector.

The fourth failure condition is dependence on arbitrary representation choice.

If the defect energy or charge depends on a gauge frame, labeling convention, basepoint choice, or equivalent groupoid representative, it does not descend to physical moduli.

The fifth failure condition is uncontrolled lattice dependence.

If the defect class disappears under harmless subdivision or the extracted energy varies without a scaling relation, the result is likely a discretization artifact.

The sixth failure condition is inserted mass.

If a local coefficient is chosen to reproduce a desired mass and the final value merely reflects that input, the theory has not derived mass.

The seventh failure condition is absence of a stable spectral signal.

If correlation functions do not show a consistent exponential decay or the transfer-matrix spectrum does not contain a distinguishable excitation, the action-gap interpretation is insufficient.

The eighth failure condition is vanishing continuum gap.

A positive finite-lattice gap may disappear under increasing volume or refinement.

In that case the excitation is not massive in the continuum limit.

The ninth failure condition is arbitrary sector engineering.

If the candidate groupoid, defect term, or boundary condition is designed specifically so that the desired charge and mass must occur, the result has little explanatory value.

The model should use simple rules and test whether nontrivial sectors emerge robustly.

The tenth failure condition is instability against allowed decay.

A proposed particle-like defect is not stable if it can decay into lower-energy states carrying the same total conserved labels.

It may still be interpreted as a resonance, but its lifetime must be calculated.

The eleventh failure condition is absence of transportability.

If a defect cannot move without losing its invariant class or encountering an infinite action barrier caused solely by the lattice, it does not provide a satisfactory model of a mobile particle.

The twelfth failure condition is nonrelativistic dispersion in a claimed fundamental regime.

A finite defect can be a legitimate excitation even with nonrelativistic dynamics, but it should not be identified with a fundamental relativistic particle unless its low-energy dispersion supports that interpretation.

The thirteenth failure condition is unconstrained proliferation.

If the model produces an enormous arbitrary collection of stable defects with no selection mechanism, it may possess less explanatory power than the theories it seeks to replace.

A ToE candidate must explain why the observed sectors are present and why others are absent or inaccessible.

The fourteenth failure condition is inconsistency between charge and defect composition.

If defect fusion violates the representation rules of the automorphism structure or total charge is not conserved under allowed dynamics, the charge interpretation fails.

The fifteenth failure condition is incompatibility with recovered gauge theory.

In the one-object limit, the defect and mass definitions should reduce to valid gauge-invariant constructions within the corresponding lattice theory.

If they conflict with known results, the broader interpretation is suspect.

These failure conditions suggest a concrete computational protocol.

First, enumerate gauge orbits and obstruction sectors on a small complex.

Second, identify minimum-action configurations in each sector.

Third, apply random gauge transformations and verify invariant classification and energy.

Fourth, perturb and relax each defect to test stability.

Fifth, enlarge the lattice and measure localization and energy scaling.

Sixth, construct gauge-invariant defect operators and calculate correlation functions.

Seventh, extract finite-size masses and compare with direct energy differences.

Eighth, move defects and determine their effective dispersion.

Ninth, combine defects and test charge fusion and annihilation.

Tenth, compare all results against null models in which topology, matter coupling, or heterogeneous identity structure is removed.

The null models are essential.

If the same localized structures appear in random configurations without the proposed obstruction mechanism, the interpretation is weakened.

If defect stability disappears when a specific structural ingredient is removed, that ingredient is causally implicated.

Useful ablations include:

remove face coherence cost;

remove matter coupling;

remove nontrivial topology;

replace the groupoid by a trivial group;

remove identity-sector variation;

remove higher cells;

or randomize the transport-composition rules.

The model should report not only positive examples but the phase diagram of success and failure.

A defect may exist only above a critical coherence coupling.

A mass gap may vanish at a phase transition.

A symmetry sector may dominate only within a finite parameter range.

These boundaries are part of the theory's predictive structure.

The scientific claim should be proportional to the result.

If the paper establishes only a gauge-invariant finite defect gap, it should say so.

If it also shows robust scaling and propagation, it may claim a particle-like excitation.

If it derives a continuum mass and charge spectrum, it may make a stronger physical claim.

If it reproduces observed particle data without parameter fitting, it would represent a major advance.

The present section establishes the criteria by which these levels can be distinguished.

The central proposal can now be stated precisely.

A physical object is represented by a stable component or structured sector of the moduli space of equivalence fields.

Its charge is the representation through which its identity responds to local automorphisms.

Its mass is the lowest invariant energy required to maintain and transport that sector through time.

Its stability follows from obstruction, conservation, and energy minimization.

Its particle-like character requires localization, transportability, and an appropriate low-energy dispersion relation.

This formulation turns the original equivalence-first ontology into a calculable research program.

The next section develops quantization over the moduli groupoid and shows how automorphism weighting, defect amplitudes, and interference among alternative identifications can be defined in a finite exact model.

8. Quantization Over the Moduli Groupoid

8.1 Finite Euclidean Partition Function

The classical action ranks admissible configurations according to their coherence cost. Quantization requires a rule for combining all physically allowed configurations while avoiding repeated counting of equivalent presentations.

Let:

M = Conf(K,G) // Gauge

denote the moduli groupoid of configurations.

Its objects are raw assignments:

Phi = ({A_v},{U_e},{psi_v}),

and its morphisms are admissible local gauge transformations between those assignments.

The physical quantum sum should be defined over this moduli structure rather than over an uncorrected set of raw configurations.

For a finite complex, finite groupoid, and finite matter-state space, the Euclidean partition function is defined by:

Z
= sum over [Phi]
exp[-S_URD(Phi) / hbar] / |Aut(Phi)|.

Here:

[Phi]

denotes an isomorphism class of configurations in the moduli groupoid, and:

Aut(Phi)

is the stabilizer group of Phi under admissible gauge transformations.

The action is constant on each gauge orbit:

S_URD(g · Phi) = S_URD(Phi).

Therefore the exponential weight is well defined on the physical equivalence class.

The factor:

1 / |Aut(Phi)|

incorporates the internal symmetry of the configuration.

This formula is the weighted groupoid cardinality of the configuration groupoid, with the Euclidean Boltzmann factor attached to each isomorphism class.

The partition function is finite when:

the complex contains finitely many cells;

the identity groupoid is finite;

the matter spaces are finite or discretized;

and the action is finite on every allowed configuration.

This exact finite setting is important because it avoids several analytical difficulties of continuum quantum field theory.

There is no formal infinite-dimensional measure.

There is no ultraviolet divergence caused by arbitrarily short wavelengths.

There is no need to choose a gauge-fixing determinant before the finite quotient structure has been understood.

Every configuration, orbit, stabilizer, and contribution can in principle be enumerated exactly.

The finite theory therefore supplies a laboratory for testing the proposed quantization rule before any continuum extrapolation is attempted.

In the one-object case with a finite gauge group G and fixed object assignment, the local gauge group is:

Gauge = G^|V|,

subject to boundary restrictions.

If every raw configuration is counted once, an equivalent expression is:

Z
= 1 / |Gauge|
sum over raw Phi
exp[-S_URD(Phi) / hbar].

To see the equivalence, partition the raw configuration set into gauge orbits.

For one orbit [Phi]:

|Orbit(Phi)|
= |Gauge| / |Aut(Phi)|.

Its contribution to the raw sum divided by |Gauge| is:

1 / |Gauge|
times
|Orbit(Phi)|
times
exp[-S_URD(Phi) / hbar]

which equals:

exp[-S_URD(Phi) / hbar] / |Aut(Phi)|.

Summing over all orbits gives the groupoid expression.

This equality provides an exact computational check.

For a sufficiently small lattice, both forms should be evaluated independently.

Their numerical agreement verifies:

correct gauge action;

correct orbit enumeration;

correct stabilizer calculation;

and correct treatment of the boundary gauge group.

The more general many-object groupoid requires additional care.

The gauge group acting on a configuration may depend on its object assignment:

Gauge(Phi)
= product over v of Aut(A_v).

Different configurations may therefore possess different ambient gauge-group orders.

One cannot always divide the entire raw sum by one universal number.

The orbit-weighted groupoid formula remains the conceptually primary expression:

Z
= sum over [Phi]
exp[-S_URD(Phi) / hbar] / |Aut(Phi)|.

If configurations can be organized into sectors with the same object assignment and the same ambient gauge group, each sector may also be evaluated through a raw sum divided by the corresponding gauge-group order.

The partition function may be decomposed by defect, charge, topology, identity sector, or boundary data.

For a defect label d:

Z_d
= sum over [Phi] with D(Phi)=d
exp[-S_URD(Phi) / hbar] / |Aut(Phi)|.

The full partition function is:

Z = sum_d Z_d.

For an automorphism-representation sector r:

Z_r
= sum over [Phi] in sector r
exp[-S_URD(Phi) / hbar] / |Aut(Phi)|.

These restricted partition functions permit calculation of:

sector probabilities;

free energies;

mass gaps;

phase competition;

and defect suppression.

At an effective temperature T, one may write the weight as:

exp[-E(Phi) / (k_B T)].

In the Euclidean quantum formulation, the weight is:

exp[-S_E(Phi) / hbar].

The two expressions are related when the Euclidean temporal extent is interpreted thermally:

beta_T = 1 / (k_B T).

Care must be taken not to confuse the gauge coupling customarily denoted beta with inverse temperature.

The paper should use distinct symbols when both appear.

For example:

beta_g

may denote the gauge-coherence coupling, while:

beta_T

denotes inverse temperature.

The Euclidean partition function has several roles.

It normalizes expectation values.

It determines the free energy:

F = -k_B T log Z

in a thermal interpretation.

It determines vacuum amplitudes in the Euclidean quantum interpretation.

It permits exact comparison among identity and defect sectors.

It also provides a nonperturbative definition of the finite theory.

The partition function is not yet a complete quantum theory.

Additional conditions are required to recover a Hilbert space and unitary time evolution.

These may include:

reflection positivity;

a positive transfer matrix;

consistent temporal composition;

and a controlled analytic continuation.

The finite Euclidean sum is nevertheless the appropriate starting point because it is mathematically well defined and directly computable.

The first exact examples should include:

a single face with a finite Abelian group;

two faces sharing an edge;

a periodic strip;

a small torus;

and a heterogeneous groupoid with two object sectors.

For each example, the calculation should report:

the number of raw configurations;

the number of gauge orbits;

the stabilizer of each orbit;

the action spectrum;

the exact partition function;

and selected observable expectations.

These examples will test whether the equivalence-first quantization differs from an ordinary quotient or uncorrected raw count.

The partition function should also be tested under relabeling of the groupoid objects and lattice vertices.

Equivalent presentations of the same finite model must produce the same Z.

This invariance is a basic consistency requirement.

The finite Euclidean partition function converts the classical coherence action into a probabilistic sum over physical relational structures.

It is the first quantitative implementation of the claim that quantum theory should combine admissible equivalences rather than multiply gauge-redundant descriptions.

8.2 Why Automorphism Weighting Matters

Automorphism weighting is not an optional normalization detail. It is required whenever configurations possess different amounts of internal symmetry.

Consider two physical configuration classes:

[Phi_1]

and:

[Phi_2].

Suppose they have equal action:

S_URD(Phi_1) = S_URD(Phi_2).

If their stabilizer groups have different sizes:

|Aut(Phi_1)| != |Aut(Phi_2)|,

then the two classes contain different numbers of raw presentations within a fixed finite gauge group.

The orbit-stabilizer relation gives:

|Orbit(Phi_i)|
= |Gauge| / |Aut(Phi_i)|.

A highly symmetric configuration has a larger stabilizer and therefore a smaller gauge orbit.

A generic configuration has a smaller stabilizer and therefore more distinct raw representatives.

If one sums over raw configurations without dividing by gauge volume, the generic orbit receives more weight solely because it has more equivalent descriptions.

That weighting would confuse presentation multiplicity with physical multiplicity.

The quotient-set approach creates the opposite problem.

If one assigns exactly one unit of weight to every gauge orbit, one ignores the internal automorphism structure carried by each physical class.

The groupoid formula occupies the correct middle position.

It removes redundancy while retaining symmetry:

physical weight of [Phi]
proportional to
1 / |Aut(Phi)|.

This is the finite analogue of integrating over a moduli space while accounting for stabilizers.

The factor can be understood in several complementary ways.

First, it follows from dividing the raw configuration sum by the local gauge-group volume.

Second, it is the natural cardinality weight of a finite groupoid.

Third, it corrects for repeated descriptions.

Fourth, it retains the fact that symmetric configurations have nontrivial self-equivalences.

Fifth, it affects entropy and free-energy comparisons among sectors.

Suppose a configuration is invariant under a nontrivial subgroup H of the gauge group.

Then:

Aut(Phi) = H.

Its groupoid weight includes:

1 / |H|.

This does not mean that symmetry is simply punished.

The total statistical preference of a sector depends on many factors:

the number of physical orbits;

their action values;

their stabilizer sizes;

their matter degeneracies;

and their topological multiplicities.

A sector with high symmetry may still dominate if it contains low-action configurations or many inequivalent states.

The automorphism factor ensures only that equivalent descriptions are not mistaken for additional physical states.

This issue becomes especially important in heterogeneous groupoids.

Different identity objects may have different automorphism groups.

Suppose sector A has:

$|\mathrm{Aut}(A)| = n_A,$

and sector B has:

$|\mathrm{Aut}(B)| = n_B.$

A naive count of raw local presentations may heavily favor the sector with the larger automorphism group.

But many of those presentations are gauge-related.

After quotienting, the physical weights may change substantially.

Therefore the dynamic selection of an automorphism sector cannot be inferred merely from the size of its symmetry group.

The complete moduli-groupoid partition function must be computed.

Automorphism weighting also affects the proposed equivalence entropy.

A naive fiber count would assign:

$S_{naive} = k_B \log N,$

where N is the number of presentations.

But if many presentations are related by self-symmetry, the physically meaningful multiplicity should use a symmetry-corrected count.

For a finite groupoid X, the homotopy cardinality is:

$$|X|_h = \sum_{[x]} \frac{1}{|\mathrm{Aut}(x)|}.$$

A corresponding entropy may be defined as:

$$S_{eq} = k_B \log |X|_h,$$

when the cardinality is positive and the thermodynamic interpretation is appropriate.

More generally, one may define sector entropies through restricted groupoid sums.

This makes entropy sensitive not only to the number of presentations but also to their self-equivalence structure.

The same correction appears in defect counting.

Suppose a defect has several equivalent placements or internal orientations.

Some of these differences may be physical degeneracies.

Others may be gauge presentations.

The orbit and stabilizer analysis determines which are which.

A physical translational degeneracy should remain in the state count.

A local gauge redundancy should not.

An internal automorphism that leaves the complete defect unchanged contributes to Aut(Phi).

A global symmetry transforming the defect into a physically different state may instead generate a genuine multiplet.

The distinction depends on which transformations are included in the gauge group and which act as physical symmetries on boundary or asymptotic data.

This must be specified explicitly.

Automorphism weighting also matters for configurations with enhanced symmetry.

At generic parameter values, a configuration may have a small stabilizer.

At a phase transition or special point, the stabilizer may become larger.

The moduli space then develops singular or orbifold-like structure.

A quotient set treats such points as ordinary elements.

The groupoid detects their enhanced self-equivalence.

This can influence:

partition functions;

semiclassical expansions;

fluctuation determinants;

and phase structure.

In a continuous compact gauge group, the finite order |Aut(Phi)| is replaced by an appropriate group volume or functional determinant treatment.

Residual gauge symmetries and zero modes require special care.

The finite model avoids these analytic complications while preserving the conceptual role of stabilizers.

It therefore provides a precise environment in which to test the physical meaning of automorphism weighting.

A computational implementation should verify several identities.

For each orbit:

|Orbit(Phi)| |Aut(Phi)| = |Gauge|,

when a common finite gauge group acts.

The sum of orbit sizes should equal the number of raw configurations.

The raw divided sum should equal the groupoid-weighted orbit sum.

Relabeling configurations should not alter stabilizer orders.

Gauge-equivalent configurations should possess isomorphic stabilizers.

These checks should be automated.

Failure of any identity indicates an error in orbit enumeration or group action.

The automorphism factor is also relevant to the ToE ambition.

If gauge groups and matter sectors are to be selected dynamically, their symmetry volumes affect their quantum weights.

A theory that compares only classical minimum actions may select one sector.

The full quantum partition function may select another because fluctuation and symmetry factors differ.

The physical vacuum should therefore be determined by the effective free energy, not by classical action alone.

The sector free energy is:

F_i
= -k_B T log Z_i

in a thermal formulation, or its Euclidean quantum analogue.

The dominant identity sector minimizes F_i, not necessarily S_min(i).

Automorphism weighting contributes directly to Z_i and hence to sector selection.

This provides a concrete mechanism by which symmetry, entropy, and coherence cost compete.

It also prevents the vague assertion that nature chooses the most symmetric configuration.

The theory instead asks:

Which moduli sector has the greatest total quantum weight after action, multiplicity, topology, and automorphism structure are all included?

That question is precise and computable in the finite model.

8.3 Observables and Correlation Functions

An observable is a gauge-invariant or coherently descending function on the moduli groupoid.

Let:

O(Phi)

be a real-valued, complex-valued, matrix-valued, or categorical quantity defined on configurations.

For a scalar expectation value, require:

O(g · Phi) = O(Phi)

for every admissible local gauge transformation g.

The Euclidean expectation value is:

For several observables:

<O_1 ... O_n>
= 1 / Z
sum over [Phi]
O_1(Phi) ... O_n(Phi)
exp[-S_URD(Phi) / hbar]
/ |Aut(Phi)|.

These correlation functions encode the statistical and quantum structure of the finite theory.

The simplest observables are functions of closed-loop holonomy.

For a loop gamma and representation rho:

W_gamma(Phi)
= Tr rho(H_gamma).

The expectation value:

<W_gamma>

measures the average response of a represented probe transported around gamma.

In a recovered lattice gauge theory, the dependence of large Wilson loops on the enclosed area or perimeter diagnoses confinement and screening behavior.

In the broader groupoid theory, loop observables also probe which identity sectors and automorphism structures are present.

Face-energy observables are:

E_f(Phi) = K(H_f).

The average face coherence cost is:

<E_face>
= 1 / |F|
sum_f <K(H_f)>.

Its fluctuations are related to response functions and possible phase transitions.

A curvature susceptibility may be defined by:

chi_K
= <(sum_f K(H_f))^2>

- <sum_f K(H_f)>^2,

with appropriate normalization.

A peak in chi_K may indicate a transition between coherence regimes.

Matter observables must be gauge invariant.

A local norm:

|psi_v|^2

is gauge invariant when the representation is unitary.

An open-path relational observable may be defined by:

R_p
= psi_t(p)^dagger rho(U_p) psi_s(p).

Under local gauge transformations, the endpoint factors cancel.

Such observables measure coherence between matter presentations at separated vertices.

A gauge-invariant matter two-point function may require a path or a sum over paths:

C(v,w)
= <psi_w^dagger rho(U_p) psi_v>.

The result may depend on p when curvature is present.

This path dependence is itself physical.

One may average over paths, minimize over path cost, or use a specified geometrically meaningful path.

The theory should state which construction is used.

Defect observables include:

the sector indicator;

the total topological charge;

the enclosed holonomy;

the localized excess action;

the defect position;

and the defect radius.

For a defect sector d, define an indicator:

I_d(Phi)
= 1 if D(Phi)=d,
= 0 otherwise.

Then the sector probability is:

P(d)
= <I_d>
= Z_d / Z.

The mean defect density is:

<n_d> / Volume,

where n_d counts defects of type d.

The defect susceptibility measures fluctuations in this number.

Charge observables may be constructed from characters of the automorphism representation.

If a configuration belongs to charge sector r, one may define:

Q_r(Phi)

through projection operators onto irreducible representations.

For a finite group G, the character projector is schematically:

P_r
= d_r / |G|
sum over g in G
chi_r(g)^*
U(g),

where d_r is the representation dimension and U(g) is the action on the state space.

The finite model can use such projectors to decompose the transfer matrix or correlation functions into charge sectors.

Mass observables are extracted from Euclidean-time correlations.

Let O_r create an excitation in sector r.

Define:

C_r(t)
= <O_r(t) O_r(0)^dagger>.

At large t:

C_r(t)
approximately equals
sum_n A_n exp[-E_(n,r) t / hbar].

The smallest E_(n,r) with nonzero overlap dominates.

The sector mass gap is:

Delta_r
= E_(0,r) - E_0.

A reliable calculation requires:

gauge-invariant operators;

sufficient temporal extent;

control of excited-state contamination;

and finite-size scaling.

The moduli-groupoid formulation affects correlation functions through the automorphism weights.

Configurations with different stabilizers contribute differently.

This can change sector probabilities and extracted effective dynamics compared with naive orbit counting.

The model should calculate selected observables in three ways where possible:

direct raw sum divided by gauge-group order;

orbit sum with automorphism weighting;

and Monte Carlo sampling.

Agreement among these methods provides strong validation.

Observables can also diagnose emergent geometry.

If a transport cost defines an effective distance d(v,w), one may examine correlation decay as a function of d rather than combinatorial edge count.

An effective correlation length can be extracted:

C(d)
approximately equals
exp(-d / xi).

The relationship:

m approximately equals hbar / (c xi)

may then be tested after an effective propagation speed c has been identified.

Spectral-dimension observables can be constructed from diffusion on the low-cost transport network.

These belong primarily to the emergent-geometry section, but the quantum expectation values defined here provide the statistical ensemble over which they are measured.

The theory should distinguish gauge-invariant observables from gauge-fixed diagnostics.

Gauge-fixed quantities may be useful for visualization and numerical acceleration.

They are not physical unless their final conclusions can be expressed in gauge-invariant form.

For example, a defect profile may appear simple in one gauge and complicated in another.

Its physical localization should be measured through gauge-invariant action density, charge density, or correlation functions.

The position of a gauge-dependent field component alone is not sufficient.

The observables also provide the bridge to falsifiability.

A theoretical interpretation becomes scientifically meaningful when it changes or constrains an expectation value, correlation function, spectrum, or scaling law.

The finite model should therefore define its claims operationally.

A stable defect exists if:

its sector probability is nonzero;

its charge is invariant;

its correlation function exhibits a persistent excitation;

and its localization survives scaling.

A mass gap exists if:

a gauge-invariant correlation function has a nonzero asymptotic decay rate that remains finite in the intended limit.

A preferred automorphism sector exists if:

its restricted partition function dominates over a robust parameter region without being forced by an arbitrary baseline term.

An emergent metric exists if:

transport costs and correlation behavior satisfy the required consistency relations.

The observable formalism turns these statements into calculations.

8.4 Real-Time Amplitudes and the Lorentzian Extension

The Euclidean partition function supplies a controlled finite quantization, but physical evolution ultimately requires amplitudes and causal composition.

The formal Lorentzian amplitude between boundary configurations Phi_i and Phi_f is:

K(Phi_f,Phi_i)
= sum over histories Phi with specified boundaries
exp[i S_L(Phi) / hbar]
divided by the appropriate automorphism factors.

More precisely, the sum should be taken over isomorphism classes of histories:

K(Phi_f,Phi_i)
= sum over [Phi]: Phi_i -> Phi_f
exp[i S_L(Phi) / hbar] / |Aut_boundary(Phi)|,

where the automorphism factor must respect fixed boundary data.

The exact definition depends on whether gauge transformations are required to be trivial on the boundary or are permitted to act on boundary states.

If boundary configurations are fixed as presentations, gauge transformations should usually restrict to the identity there.

If boundary states are themselves physical equivalence classes, one must quotient boundary gauge symmetry consistently.

The finite model allows these choices to be stated explicitly.

A history is not merely one static configuration.

It is an assignment on a complex containing a distinguished temporal direction or a cobordism-like relation between initial and final boundaries.

The interior variables include:

identity objects;

spatial and temporal edge transports;

face holonomies;

matter data;

and possibly defect worldlines.

The Lorentzian action S_L differs from the Euclidean action by signature-dependent signs and possible factors of i.

It should not be obtained by informal substitution alone.

The theory must define either:

a direct discrete Lorentzian action;

or a justified analytic continuation from the Euclidean model.

The real-time sum is oscillatory.

Contributions with different action phases can interfere constructively or destructively.

This interference is central to the proposed interpretation of quantum behavior as interference among admissible identifications.

However, the Lorentzian formulation introduces several difficulties.

The amplitude may suffer severe cancellation.

A naive numerical sum may be unstable.

Unitarity is not automatic.

Causal structure is not yet derived.

The relationship between the combinatorial temporal direction and physical time must be justified.

The first constructive paper should therefore treat the Lorentzian form as a defined extension and demonstrate it only in small exact examples.

A minimal temporal complex may contain:

an initial slice;

one intermediate layer;

and a final slice.

All histories connecting specified boundary data can be enumerated.

The amplitude can then be calculated exactly.

Composition should be tested.

If an intermediate boundary Phi_m is inserted, the amplitudes should satisfy:

K(Phi_f,Phi_i)
= sum over physical Phi_m
K(Phi_f,Phi_m)
K(Phi_m,Phi_i)

with the correct measure and automorphism weighting.

This is the discrete gluing law.

It expresses the categorical principle that physical processes compose.

The measure over intermediate states must be chosen so that repeated gauge-equivalent boundary presentations are not overcounted.

The groupoid structure therefore enters temporal composition directly.

A transfer matrix provides a practical bridge between Euclidean and Lorentzian theories.

Let T propagate the state by one Euclidean time step.

Its matrix elements are defined by summing over configurations in one temporal slab:

T(Phi',Phi)
proportional to
sum over interior slab variables
exp[-S_slab / hbar],

with gauge constraints and automorphism factors included.

If T is positive and self-adjoint under the correct inner product, one may define a Hamiltonian:

T = exp(-a_t H / hbar).

The quantum state space is then constructed from gauge-invariant boundary configurations or functions on the boundary moduli groupoid.

Real-time evolution is:

U(t) = exp(-i H t / hbar).

This route is preferable to beginning with an unconstrained Lorentzian sum because it makes the questions of positivity, spectrum, and gauge invariance explicit.

The Hilbert space should not consist of arbitrary functions on raw edge assignments unless gauge constraints are imposed.

It should consist of gauge-invariant states or an equivalent quotient construction.

For a finite boundary configuration groupoid B, one may consider complex-valued functions on its isomorphism classes with an inner product weighted by automorphism factors.

A schematic inner product is:

<psi|phi>
= sum over [b]
psi([b])^*
phi([b])
/ |Aut(b)|.

The precise normalization should be derived from the gluing law.

This provides a finite model of a quantum state space built directly from physical equivalence classes.

The Lorentzian extension also allows defect propagation to be studied.

A defect present on the initial boundary may appear at a different spatial position on the final boundary.

The amplitude sums over all admissible histories carrying its invariant charge and obstruction class.

A defect worldline is then a sequence of localized coherence patterns connected across temporal slices.

The particle is not represented by one fixed field configuration.

It is represented by the coherent propagation of a physical sector.

The effective propagator may be extracted from:

K_d(x_f,t_f;x_i,t_i).

At large scales, one can test whether it approaches a relativistic propagator or another effective form.

The real-time extension is therefore where mass as equivalence inertia becomes dynamical.

A nonzero coherence cost for maintaining the defect through time generates phase accumulation:

exp[-i E_d t / hbar].

Motion changes the action and hence the propagator.

The finite model can test whether the resulting dispersion relation approximates:

E^2 = m^2 c^4 + p^2 c^2.

Until such a result is obtained, the model has a defect energy but not yet a demonstrated relativistic particle mass.

8.5 Interference Among Identifications

The central interpretive proposal of the quantum theory is that amplitudes sum not merely over trajectories of primitive objects but over admissible relational identifications connecting physical boundary conditions.

Suppose an initial physical presentation x and a final physical presentation y can be connected by several histories:

e_1, e_2, ..., e_n.

Each e_i represents a coherent way of transporting, deforming, or reconstructing physical identity from the initial context to the final context.

The amplitude is:

K([x],[y])
= sum_i A(e_i),

with:

A(e_i)
= a_i exp[i S(e_i) / hbar].

The probability associated with an appropriate measurement is not generally the sum of the individual probabilities:

sum_i |A(e_i)|^2.

Instead:

|sum_i A(e_i)|^2

contains cross terms:

A(e_i) A(e_j)^*.

These cross terms represent interference among distinct identifications.

The same initial and final physical meanings may be related in multiple coherent ways.

Quantum behavior arises when those ways remain simultaneously accessible and retain meaningful relative phase.

This interpretation is compatible with ordinary path-integral quantum mechanics but shifts the ontological emphasis.

The conventional language says that a particle takes all paths in the amplitude sum.

The equivalence-first language says that the theory sums all admissible ways of identifying the preparation with the outcome.

The difference is subtle but important.

A path need not represent the literal trajectory of a preexisting object.

It may represent one morphism or history through which physical identity is transported.

The finite groupoid model allows this proposal to be tested exactly.

Consider two distinct histories h_1 and h_2 between the same boundary configurations.

Their contributions are:

A_1 = exp[i S(h_1) / hbar],

A_2 = exp[i S(h_2) / hbar].

The combined probability contains:

2 cos[(S(h_1)-S(h_2))/hbar].

If the action difference changes, the interference pattern changes.

If the histories become distinguishable through an environmental record, their cross terms may be suppressed.

The histories may differ in:

edge transports;

face holonomies;

defect worldlines;

identity-object sequences;

topological classes;

or higher coherence data.

Not every difference produces physically distinct alternatives.

Histories related by a gauge transformation represent the same physical identification and should not be counted as separate interfering alternatives.

The amplitude sum must therefore be taken over physical history classes, with automorphism correction.

This is a critical advantage of the moduli-groupoid formulation.

It distinguishes:

multiple presentations of one history;

from multiple physically inequivalent histories.

Only the latter contribute distinct amplitude terms.

Failure to make this distinction would produce spurious interference caused by gauge redundancy.

The simplest exact interference experiment in the finite model should mimic a two-path system.

Construct a complex with two inequivalent routes between initial and final boundary vertices.

Assign transports and actions to both routes.

Calculate the physical amplitude.

Then introduce a gauge transformation and verify that the interference probability remains unchanged.

Next, introduce nontrivial holonomy between the paths.

The relative phase should depend on the enclosed flux or groupoid holonomy.

This reproduces an Aharonov-Bohm-like structure in the finite model.

The interpretation is direct:

the two identification routes differ by a closed loop;

the loop carries nontrivial identity holonomy;

the relative phase detects that holonomy.

A non-Abelian version would require path-ordered transport and internal-state interference.

The amplitudes become matrices or operators acting on representation spaces.

The interference then depends on the ordered product of noncommuting transformations.

This provides a richer test of charge as automorphism response.

The defect sector offers another interference experiment.

A defect may propagate from x_i to x_f along several worldline histories.

If the histories have the same boundary charge and remain coherent, their amplitudes interfere.

If one history winds around a topological flux, it may acquire a different phase or internal transformation.

The resulting interference measures the relational topology.

Higher morphisms may organize when two histories are themselves equivalent.

Suppose:

alpha : h_1 => h_2

is a 2-morphism.

The theory must determine whether h_1 and h_2 are:

two separate amplitude contributions;

two gauge-related presentations of one contribution;

or two coherently related contributions whose relation supplies an additional phase or weight.

The answer depends on which level of structure is treated as physical and which is quotiented.

This is one reason the 1-groupoid model is only the first stage.

At the 1-truncated level, distinct morphisms not related by gauge transformation contribute separately.

At the higher level, 2-equivalence may further identify or organize them.

The finite model can approximate this by explicitly specifying equivalence relations among histories.

A future higher-groupoid formulation should derive the rule from the complete moduli object.

The interference interpretation also applies to superposition.

A superposed quantum state can be understood as retaining several physically admissible identification channels before a measurement or environment renders them distinguishable.

The state is not simply ignorance about which path occurred.

The relative phases remain operationally accessible.

When all relative phase information is lost, the alternatives behave like a classical mixture.

This leads directly to decoherence and truncation.

The theory should distinguish three operations.

Coherent summation:

A_total = sum_i A_i.

Classical probabilistic mixing:

P_total = sum_i p_i P_i.

Gauge quotienting:

several descriptions are treated as one physical alternative.

These are not the same.

Gauge quotienting removes redundant presentations before amplitudes are summed.

Coherent summation combines distinct physical alternatives.

Decoherence converts some coherent sums into effective mixtures by suppressing cross terms.

A correct equivalence-first quantum theory must preserve all three distinctions.

The proposal can fail in several ways.

If physical history classes cannot be defined independently of presentation, the amplitude sum is ambiguous.

If automorphism weighting destroys temporal composition, the measure is incorrect.

If the finite model cannot reproduce standard two-path interference in the recovered gauge sector, the interpretation is inadequate.

If all alternative identifications can be reduced to ordinary paths without additional explanatory or predictive value, the framework remains a reinterpretation rather than a new theory.

If the higher equivalence structure yields a new restriction, topological phase, or selection rule, the proposal gains distinct physical content.

The initial goal is therefore not to claim a new quantum mechanics.

It is to demonstrate that quantum amplitudes can be consistently formulated on the moduli groupoid of equivalence histories and that ordinary interference is recovered without gauge overcounting.

8.6 Measurement as Truncation

The premeasurement quantum structure retains multiple configurations, paths, phases, and possibly higher relations among those paths.

A classical measurement produces a comparatively restricted record.

The theory proposes that this transition can be modeled as a truncation of relational structure.

Let R denote the higher moduli object describing the quantum alternatives accessible before measurement.

At minimum, R contains:

physical configurations;

equivalences among configurations;

alternative histories;

and phase-weighted relations among those histories.

In a full Homotopy Type Theory interpretation, R may contain higher identity paths at all levels.

A 0-truncation:

tau_0 : R -> ||R||_0

retains only connected components.

In geometric language, this is analogous to passing to:

pi_0(R).

The detailed paths connecting presentations within a component are forgotten.

The result is set-like.

A 1-truncation:

tau_1 : R -> ||R||_1

retains objects and ordinary equivalences while discarding nontrivial higher equivalences between paths.

Higher truncations preserve progressively more relational structure.

The measurement hypothesis is that an actual experimental process may map a rich relational object into a lower-truncated outcome structure.

A classical detector record may distinguish only a finite set of macroscopically stable components:

Outcome = {o_1,o_2,...,o_n}.

The measurement map is then schematically:

M : R -> Outcome.

For M to be physical, it must descend through gauge equivalence.

Gauge-related presentations must produce the same recorded outcome.

Thus:

Phi equivalent to Phi'
implies
M(Phi) = M(Phi').

Measurement does not select among gauge representatives.

It selects among physically distinct outcome sectors.

This distinction is fundamental.

A gauge choice is not a measurement outcome.

The finite model permits a precise first construction.

Let M_conf be the configuration groupoid.

Let:

pi_0(M_conf)

be its set of connected physical sectors under a specified class of accessible coherence relations.

Define:

tau_0(Phi)
= the connected component containing Phi.

This map forgets:

the specific gauge representative;

the transformations connecting representatives;

the stabilizer structure;

and higher paths within the component.

If the outcome apparatus records only the component label, the observation is a 0-truncated image of the full configuration structure.

However, this formal truncation alone does not solve the quantum measurement problem.

Several questions remain.

Why does one outcome occur in an individual experiment?

How are outcome probabilities determined?

Why do probabilities follow the Born rule?

What physical interaction causes the effective loss of higher-path accessibility?

Why are some observables selected as stable pointer quantities?

How does an irreversible record form?

The theory must not claim that writing:

tau_0 : R -> pi_0(R)

answers these questions by itself.

It provides a mathematical form for the loss of relational information.

A physical measurement model requires dynamics.

Let the system S interact with an apparatus A and environment E.

Initially, the system may contain coherent alternatives:

sum_i c_i |s_i>.

The measurement interaction correlates them with apparatus states:

sum_i c_i |s_i>|a_i>.

Environmental interaction produces:

sum_i c_i |s_i>|a_i>|e_i>.

If the environmental states become effectively orthogonal, interference between distinct apparatus records becomes inaccessible to local observation.

In the equivalence-first language, the higher paths connecting the outcome branches are dispersed into system-apparatus-environment relations that the observer cannot coherently control.

The accessible object becomes effectively truncated.

The full relational structure may remain mathematically present, but the experimentally available map retains only stable components.

This gives a possible interpretation of decoherence:

decoherence is the loss of accessible transformations comparing alternative outcome presentations.

The phrase accessible is essential.

The theory need not assert that all higher paths are metaphysically destroyed.

It may assert only that the measurement context no longer supports operations capable of revealing their relative phase.

A finite model can represent this process through a quantum channel.

Let rho be a density operator on the finite physical state space.

A decohering channel:

D(rho)

suppresses off-diagonal terms between selected outcome sectors.

The truncation map can be represented operationally by:

rho
-> sum_i P_i rho P_i,

where P_i projects onto outcome sector i.

This is a standard dephasing structure.

The new interpretation is that the sectors correspond to components or lower-truncated images of the relational moduli object.

The channel should be derived from coupling to an explicit finite environment rather than inserted without explanation.

A minimal simulation may use:

a small gauge or groupoid system;

two physically distinct history sectors;

one apparatus variable;

and several environmental variables.

The system-apparatus coupling correlates the alternatives with distinct apparatus states.

The environment then reduces the measurable interference.

The calculation should track:

the off-diagonal coherence;

the mutual information;

the sector probabilities;

the stability of the apparatus record;

and the gauge invariance of the entire procedure.

The truncation interpretation is successful only if the outcome classification is independent of gauge presentation.

The apparatus must respond to gauge-invariant observables such as:

loop holonomy class;

defect charge;

sector membership;

or another descending quantity.

A detector that responds to a raw edge variable U_e would generally measure a presentation-dependent quantity and would not define a physical observation.

The measurement map must therefore satisfy the observable-descent axiom.

The probability problem remains central.

Given an initial quantum state:

|Psi> = sum_i c_i |i>,

standard quantum theory assigns:

$P(i) = |c_i|^2$.

The truncation map identifies the outcome sectors but does not by itself derive these weights.

The groupoid path sum determines amplitudes.

A further argument is required to show why observed frequencies equal squared amplitude magnitudes.

Possible routes include:

a measure on branches or equivalence classes;

decision-theoretic consistency;

envariance;

decoherent histories;

or a structural theorem about composition and normalization.

The present paper should identify the Born rule as an unresolved requirement unless it is actually derived.

The finite model can nevertheless test whether standard probabilities are recovered when the ordinary quantum rules are applied to the groupoid-based amplitude.

The distinction should be explicit:

the moduli-groupoid sum defines amplitudes;

decoherence suppresses interference between outcome sectors;

truncation defines the accessible classical outcome structure;

the Born rule assigns probabilities.

Only the first three are structurally addressed here.

The fourth remains a separate foundational problem.

Measurement also introduces irreversibility.

Gauge transformations are invertible.

Unitary quantum evolution is invertible.

A classical record appears effectively irreversible because reconstructing the lost phase relations requires control over an enormous environment.

In the finite model, exact reversibility may remain possible in principle.

Effective irreversibility appears when:

environmental degrees of freedom are ignored;

the observer applies a coarse-graining channel;

or only a truncated outcome algebra remains accessible.

Thus the arrow from R to pi_0(R) is not necessarily an invertible physical equivalence.

It is a noninvertible process.

This supports the broader claim that reality should be modeled not only as an infinity-groupoid of reversible equivalences but as an infinity-category containing noninvertible processes such as:

measurement;

decay;

coarse-graining;

and record formation.

The truncation map is one example of such a morphism.

The finite theory should verify several properties.

First, gauge compatibility:

tau_0(g · Phi) = tau_0(Phi).

Second, idempotence:

tau_0(tau_0(R)) = tau_0(R),

in the appropriate sense.

Third, information loss:

distinct paths or phases in R map to the same truncated outcome.

Fourth, stability:

small perturbations within one outcome basin do not change the recorded component.

Fifth, noninvertibility:

the original higher relational data cannot be reconstructed from the truncated outcome alone.

Sixth, compositional consistency:

successive measurements or coarse-grainings compose coherently.

The measurement proposal can be falsified or weakened if:

no physically natural truncation corresponds to observable outcome sectors;

the truncation depends on gauge choice;

the model cannot generate stable records;

decoherence does not suppress interference in the expected limit;

or probability weights remain arbitrary.

A successful result would not yet solve every interpretation of quantum mechanics.

It would show that classical set-like outcomes can arise as lower-information images of a richer equivalence structure through a definite physical process.

The complete quantum picture developed in this section is therefore:

the moduli groupoid identifies gauge-equivalent presentations;

the Euclidean partition function weights physical configuration classes;

automorphism factors correct symmetry counting;

observables descend to the quotient;

Lorentzian histories generate amplitudes;

distinct physical identifications interfere;

environmental coupling suppresses access to relative phase;

and measurement produces a truncated stable record.

This framework converts several broad claims of Univalent Relational Dynamics into finite mathematical constructions.

The next section uses the same partition function to investigate whether the identity object and its automorphism group can themselves be selected dynamically rather than fixed in advance.

9. Dynamical Selection of Gauge Structure

9.1 The Central Problem

The one-object reduction demonstrates that ordinary lattice gauge theory appears when every local identity object belongs to one fixed type A and every transport morphism is an automorphism of that type. The resulting gauge group is:

$G = \mathrm{Aut}(A)$.

This construction recovers gauge dynamics once A has been selected. It does not explain why nature should select that identity object or its automorphism group.

A candidate foundational theory must distinguish two questions.

The first question is:

Given a gauge group G, what fields, holonomies, defects, and quantum amplitudes follow?

Ordinary gauge theory answers this question with extraordinary success.

The second question is:

Why is the physically realized local self-equivalence structure G rather than some other group?

This is the gauge-selection problem.

In most conventional theories, the gauge group is an input. The Standard Model begins with a structure closely related to:

$SU(3) \times SU(2) \times U(1)$,

together with specified matter representations and symmetry-breaking fields. The theory then predicts interactions and scattering behavior once its parameters are fixed.

A deeper theory should seek to explain why this gauge structure, representation content, and breaking pattern occur.

Univalent Relational Gauge Dynamics reframes the problem.

A gauge group is not initially treated as a free-standing algebraic object imposed on matter. It is the automorphism group of a local identity structure:

$G = \mathrm{Aut}(A)$.

The selection of a gauge group therefore becomes the selection of an identity object or identity sector whose self-equivalences form G.

This reframing does not solve the problem automatically.

If the model simply chooses an object A with:

Aut(A) = SU(3) x SU(2) x U(1),

then the observed gauge group has merely been renamed.

The theory becomes explanatory only if a broader space of possible identity structures is defined by simple principles and the physically realized sector emerges through the dynamics or quantum weighting.

The central question is therefore:

Can the coherence action and moduli-groupoid measure select a particular automorphism structure from a family of possible identity sectors?

Several distinctions are necessary.

A connected groupoid cannot generally contain genuinely different gauge groups at different objects in the sense required for fundamental selection.

If objects A and B are connected by an invertible morphism:

u : A -> B,

then their automorphism groups are isomorphic:

Aut(B) is isomorphic to Aut(A),

through:

g -> u g u^-1.

The two groups may be represented differently, but they possess the same abstract structure.

Thus several objects within one connected groupoid component normally represent different presentations of one reversible identity sector, not genuinely different gauge-group candidates.

To compare nonisomorphic automorphism groups, the theory must include:

different disconnected groupoid components;

different groupoids within a larger family;

a higher category whose objects are groupoids or identity theories;

or noninvertible processes connecting distinct identity sectors.

This point is important because otherwise apparent gauge-group diversity may be only a relabeling of one group.

Let the candidate identity space contain components:

C_1, C_2, ..., C_N.

Choose an object A_i in each component.

The corresponding automorphism group is:

G_i = Aut(A_i).

Within one component, other object presentations possess groups isomorphic to G_i.

Between different components, the groups may be nonisomorphic.

The gauge-selection problem becomes the comparison of the physical weights of these components.

The theory must determine:

which sectors admit low-action transport;

which support stable defects;

which permit consistent matter representations;

which possess anomaly-free quantum structure;

which yield acceptable continuum limits;

and which dominate the full partition function.

The selection need not be absolute at every scale.

Different automorphism sectors may dominate in different regimes.

At high coherence energy, a larger symmetry may be realized.

At low energy, a reduced subgroup or different effective sector may dominate.

Gauge structure may therefore be scale dependent.

The theory should distinguish:

fundamental identity symmetry;

vacuum symmetry;

effective low-energy symmetry;

and residual symmetry after matter condensation.

The fundamental candidate space contains all allowed identity sectors.

The vacuum selects a particular sector or mixture.

Matter fields may further reduce the symmetry.

Coarse-graining may produce an effective group different from the microscopic automorphism structure.

These levels should not be conflated.

The gauge-selection problem is more demanding than ordinary spontaneous symmetry breaking.

In spontaneous symmetry breaking, the group G is already given. A field configuration selects a vacuum invariant under a subgroup H.

The theory explains:

G -> H,

but it does not explain the origin of G.

Dynamical identity selection asks whether G itself can emerge from competition among candidate automorphism sectors.

A successful model may combine both mechanisms:

first select G = Aut(A);

then select a vacuum whose stabilizer is H subset G.

This creates a two-stage hierarchy:

identity-sector selection;

followed by symmetry breaking within the selected sector.

The present section develops a finite framework for testing the first stage.

It does not claim to derive the observed Standard Model group.

Its immediate purpose is to determine whether gauge-group selection is a mathematically coherent and computationally testable process.

The minimal requirements are:

a nonarbitrary candidate set of identity sectors;

a universal or simply parameterized coherence action;

a consistent quantum measure;

and a selection result not forced by explicit sector-specific tuning.

Without these conditions, the model merely chooses its answer in advance.

9.2 Heterogeneous Identity Groupoid

The simplest finite implementation begins with a disjoint union of connected groupoids:

G_total
= G_1 disjoint union G_2 disjoint union ... disjoint union G_N.

Each component G_i represents one reversible identity sector.

Choose a representative object A_i in G_i and define:

G_i^aut = Aut(A_i).

To avoid confusing the groupoid component with its automorphism group, write:

C_i = connected component,

and:

G_i = Aut(A_i).

A local object assignment selects:

A_v in Ob(G_total).

An edge transport:

U_e : A_s(e) -> A_t(e)

exists only when the endpoint objects belong to the same connected component, unless additional interface structure is introduced.

In the strict reversible model, this implies that every connected region of the lattice carrying ordinary edge transport lies within one identity component.

If the full complex K is connected and every edge must carry an invertible transport, then all vertex objects must belong to the same component C_i.

The configuration therefore has a global sector label:

sigma(Phi) = i.

This first model describes global competition among identity sectors.

The partition function decomposes as:

Z = sum_i Z_i,

where Z_i sums all configurations valued in component C_i.

A more general model permits different components in different spatial regions.

Then an edge joining components C_i and C_j cannot carry an ordinary groupoid isomorphism.

The interface requires additional data.

Several approaches are possible.

The first is to forbid such mixed assignments.

This restricts the theory to globally uniform identity sectors.

The second is to assign infinite action to incompatible edges.

This produces the same effective restriction.

The third is to introduce a finite interface cost and a noninvertible transition morphism:

T_ij : A_i -> A_j.

Such morphisms belong not to the reversible groupoid but to a larger category of physical processes.

The fourth is to introduce a bimodule, profunctor, correspondence, or domain-wall object connecting the two sectors.

The fifth is to embed both sectors in a larger identity structure containing reversible or higher relations between them.

The initial gauge-selection experiment should use globally uniform sectors.

This permits exact comparison without prematurely introducing irreversible sector conversion.

The mixed-domain model can then be added to study phase boundaries and symmetry-changing transitions.

For each component C_i, define its field configurations:

Phi_i = ({A_v},{U_e},{psi_v}) with A_v in C_i.

The corresponding action is:

S_i(Phi_i)
= S_face,i

- S_matter,i
- S_identity,i
- S_boundary,i
- S_defect,i.

The action should be defined by common rules across sectors.

For example, the face cost may use a uniform class-function prescription:

K_i(h)
= 1 - Re chi_i(h) / d_i,

where chi_i is the character of a specified natural or minimal faithful representation and d_i is its dimension.

The use of different arbitrary functions K_i for different groups would permit unrestricted tuning.

A selection result is more meaningful when the same structural rule generates every sector action.

Likewise, the local identity cost M(A_i) should be either:

identical across candidate sectors;

derived from an independent complexity rule;

or varied transparently as a parameter.

If one assigns a much lower baseline cost to the desired sector, its dominance is trivial.

A neutral first comparison should set:

M(A_i) = 0

or the same constant for every sector.

The sectors then compete through:

their transport configuration spaces;

their holonomy costs;

their automorphism weights;

their matter compatibility;

their defect spectra;

and their quantum fluctuations.

The candidate set of groups must also be principled.

An arbitrary list of groups gives an arbitrary selection problem.

Several finite research strategies are possible.

One may compare all finite groups up to a chosen order.

One may compare automorphism groups arising from a specified family of finite structures.

One may compare groups generated by a fixed number of generators and relations subject to bounded complexity.

One may compare stabilizer groups of objects produced by GenesisBoard.

One may compare compact Lie groups satisfying specified dimensional, representation-theoretic, or anomaly constraints.

The first computational study should use all groups in a small finite class rather than a hand-selected pair.

For example, compare the finite groups of order less than or equal to a chosen bound.

For each group G_i, construct the one-object component BG_i.

The total candidate groupoid is:

G_total
= disjoint union_i BG_i.

Each sector has one object whose automorphism group is G_i.

This eliminates within-component presentation multiplicity and isolates the effect of different automorphism structures.

A later study can replace BG_i with richer connected groupoids containing multiple equivalent object presentations.

This will test whether presentation-space multiplicity changes sector weighting.

Matter may be included through a sector-dependent representation category.

However, allowing arbitrary matter representations in every sector introduces another large choice.

A controlled comparison may use:

the regular representation;

the smallest faithful representation;

all irreducible representations with specified weighting;

or representations generated by one universal construction.

The regular representation exists for every finite group and therefore provides a uniform benchmark.

Its dimension equals |G_i|, which may itself bias the action and state count.

This bias is not necessarily illegitimate, but it must be understood.

The smallest faithful representation may be more economical, but it is not unique in every case.

The theory should report how selection depends on this choice.

A pure gauge comparison should precede the matter-coupled comparison.

In the pure gauge model, sector differences arise from:

group multiplication;

conjugacy structure;

representation characters used in K;

number of link assignments;

number of gauge transformations;

and the distribution of plaquette holonomies.

This is already a nontrivial statistical competition.

The heterogeneous candidate space may also include identity objects with continuous automorphism groups.

Exact enumeration then becomes impossible.

The initial finite study should remain with finite groups.

If a robust selection principle is found, its extension to compact Lie groups can be investigated through character expansions, Monte Carlo integration, or discretized approximations.

The heterogeneous identity construction therefore creates a finite ensemble of possible gauge theories inside one larger model.

The question is not which fixed-group theory has been chosen by the investigator.

The question is which identity sector receives the dominant physical weight under common dynamical rules.

9.3 Competition Among Sectors

For each identity sector i, define the restricted partition function:

Z_i
= sum over [Phi_i]
exp[-S_i(Phi_i) / hbar]
/ |Aut(Phi_i)|.

The total partition function is:

Z = sum_i omega_i Z_i,

where omega_i is a prior sector weight.

A maximally neutral finite comparison sets:

omega_i = 1

unless a principled measure determines otherwise.

The probability of sector i is:

P_i = omega_i Z_i / Z.

At finite temperature, the sector free energy is:

F_i = -k_B T log(omega_i Z_i).

The dominant sector minimizes F_i.

In the zero-temperature or semiclassical limit, the leading contribution is controlled by the minimum energy or Euclidean action:

F_i approximately equals E_min,i

plus quantum and entropic corrections.

A sector may dominate for several reasons.

It may possess a lower minimum coherence action.

It may possess more low-action physical configurations.

It may support matter fields with lower transport cost.

It may possess favorable defect or boundary structure.

It may have smaller automorphism penalties.

It may generate a larger entropy of inequivalent low-cost presentations.

It may exhibit a more stable continuum limit.

The total selection is therefore not reducible to symmetry-group size.

The decomposition:

$F_i = E_i - T S_i$

is useful conceptually.

The energy term favors low coherence cost.

The entropy term favors large physical multiplicity.

The groupoid correction alters the entropy by weighting configurations according to their stabilizers.

At low temperature, energy may dominate.

At higher temperature, a sector with many low-cost configurations may become preferred even if its absolute minimum is higher.

This creates the possibility of gauge-sector phase transitions.

Suppose sectors i and j satisfy:

$F_i(T_c) = F_j(T_c)$

at a critical temperature T_c.

Below T_c, sector i dominates.

Above T_c, sector j dominates.

The effective gauge structure then changes with thermal or coherence scale.

In a cosmological interpretation, the early universe could occupy one identity sector and later transition into another.

Such an interpretation requires a real-time transition mechanism and should not be inferred solely from Euclidean free-energy crossing.

The finite model can nevertheless identify the thermodynamic possibility.

The pure gauge partition function for each finite group can be calculated exactly on small complexes.

Consider a one-face complex.

Every edge assignment produces one face holonomy h.

After gauge quotienting, the physical contributions depend on conjugacy classes.

The partition function can often be written using group characters or direct enumeration.

Larger complexes introduce correlations among plaquettes and global cycles.

The sector dependence then reflects nontrivial group structure.

The theory should compute:

Z_i;

mean face action;

holonomy distribution;

specific heat;

Wilson-loop expectations;

stabilizer distribution;

and defect-sector probabilities.

A useful normalized comparison must account for the fact that groups of different order produce different raw numbers of edge assignments.

The groupoid quotient removes local gauge redundancy but does not necessarily remove all physically meaningful differences in state-space size.

A larger group may genuinely support more inequivalent holonomies.

Whether this multiplicity should favor that sector is part of the physical hypothesis.

The theory may need an additional normalization if the comparison otherwise rewards arbitrarily large groups solely because they possess more elements.

Several possibilities exist.

One may use equal prior weight per group.

One may penalize algebraic complexity.

One may normalize by representation dimension.

One may derive a measure from a higher category of identity structures.

One may impose a finite complexity cutoff and study sensitivity to it.

No normalization should be hidden.

The selection result must report how it depends on the measure over candidate sectors.

The problem resembles model selection.

A more complex sector may fit more configurations but should perhaps carry a complexity cost.

A natural candidate is a structural action:

S_structure(G_i),

added to the sector weight:

omega_i
= exp[-S_structure(G_i) / hbar].

Possible structural costs include:

number of generators;

number of relations;

group order;

representation dimension;

description length;

or categorical construction depth.

However, each choice introduces a new hypothesis.

The first study should present results both with and without such penalties.

The competition should also be examined at fixed physical conditions.

If different sectors use different coupling normalizations, direct free-energy comparison may be meaningless.

The couplings should be related through a common prescription.

For example, choose the face class function so that its quadratic expansion near the identity has the same normalization across continuous groups.

For finite groups, choose a uniform maximum face cost or normalize by representation dimension.

The lattice, boundary conditions, and temperature should be identical.

Matter comparison requires equivalent field content rules.

The theory should distinguish intrinsic sector selection from environmental selection.

Intrinsic selection occurs when one sector dominates under neutral boundary conditions.

Environmental selection occurs when external matter, topology, or boundary data favor one group.

Both may be physically relevant.

A universe containing particular matter representations may stabilize a gauge sector that would not dominate in the pure gauge vacuum.

But if the matter content is itself inserted, the gauge selection is conditional rather than fundamental.

A deeper model should select gauge and matter structures jointly.

This can be formulated through a combined partition function:

Z_i
= sum over gauge fields, matter representations, and matter configurations
exp[-S_i / hbar]
/ automorphism weights.

The sum over representations requires a principled measure.

This lies beyond the first finite model but is necessary for the full ToE program.

The sector competition should also include anomaly and consistency constraints.

A sector with low action but inconsistent quantum matter content must be excluded or assigned zero physical weight.

Potential filters include:

gauge anomaly cancellation;

global anomaly cancellation;

unitarity;

reflection positivity;

locality;

causal consistency;

and existence of an acceptable continuum limit.

These are not arbitrary preferences.

They are necessary conditions for a viable physical theory.

The selection process may therefore have two stages.

First, consistency eliminates inadmissible sectors.

Second, the action and quantum measure rank the remaining sectors.

The set of candidate groups may shrink dramatically under these constraints.

The finite model can begin with the second stage while acknowledging that continuum consistency filters are not yet implemented.

The central computational output should be a sector phase diagram.

For couplings:

beta_g, kappa, eta, and temperature T,

plot or tabulate which sector minimizes free energy.

Determine whether the dominance is:

robust over a region;

confined to a fine-tuned point;

or absent.

A sector that dominates only after precise parameter adjustment does not provide strong explanatory selection.

A sector that dominates throughout a broad region under common rules is more significant.

The calculation should also report near-degenerate sectors.

A small free-energy difference may permit transitions, coexistence, or sensitivity to boundary conditions.

Such near-degeneracy may be physically relevant but weakens claims of unique selection.

The ideal result is not necessarily one universally dominant group.

It may be a small set of dynamically viable groups satisfying common structural constraints.

That would still be progress if it reduces the enormous space of possible gauge structures without inserting the answer.

9.4 Spontaneous Gauge Selection and Symmetry Breaking

A sector is dynamically selected when its contribution dominates the physical partition function or ground-state energy without being uniquely imposed in the definition of the theory.

Let sector i satisfy:

P_i approximately equals 1

in a specified limit or parameter regime.

Then the effective local gauge group is:

G_eff = Aut(A_i).

This is identity-sector selection.

The selected vacuum may still possess further internal structure.

Suppose matter fields in representation V_i acquire a configuration:

psi_0.

The subgroup preserving the vacuum is:

H_i
= {g in G_i | rho_i(g) psi_0 = psi_0}.

The effective low-energy symmetry becomes:

G_i -> H_i.

This is ordinary spontaneous symmetry breaking within the selected identity sector.

The complete process is therefore:

candidate identity sectors {C_i}
-> selection of C_i
-> automorphism group G_i
-> vacuum selection
-> residual group H_i.

The distinction between the two arrows is conceptually important.

The first selects which type of local identity exists.

The second selects a state or phase within that identity type.

A theory that derives only H_i from a preselected G_i has not solved the gauge-origin problem.

A theory that selects G_i but not the vacuum structure remains incomplete.

The two mechanisms may also occur together.

An order parameter may influence which identity object is stable.

Conversely, the selected identity group may determine which order parameters can exist.

A coupled action can represent this feedback.

Let sigma_v label the identity sector.

Let psi_v be a matter or condensate variable.

The local action may include:

M(sigma_v);

V_(sigma_v)(psi_v);

transport terms allowed within sigma_v;

and interface costs between different sigma values.

At high temperature, the partition function may explore many sigma configurations.

At low temperature, one sector and one matter vacuum may dominate.

The gauge structure and symmetry-breaking pattern then emerge jointly.

Mixed spatial domains require a larger process category because sectors with nonisomorphic automorphism groups are not connected by ordinary gauge morphisms.

Introduce domain-wall data:

W_ij

between sectors i and j.

The interface action is:

S_wall
= tau_ij times Area_ij

in a simple model, where tau_ij is the wall tension.

A domain wall is stable when boundary conditions or vacuum degeneracy require coexistence.

A transition may proceed through nucleation of a bubble of one identity sector inside another.

The Euclidean action of the critical bubble controls the tunneling or thermal transition rate.

This gives a possible cosmological dynamics of gauge selection.

However, such a model is substantially more complex than global sector comparison.

The initial paper should define it as an extension rather than claiming a completed early-universe scenario.

The term spontaneous gauge selection must also be used carefully.

Gauge symmetry is often regarded as descriptive redundancy rather than an ordinary physical symmetry that can literally break.

In standard gauge theory, what is commonly called gauge-symmetry breaking is more precisely a change in the vacuum and physical excitation spectrum, while gauge redundancy remains.

The present framework does not change this requirement.

Local presentation equivalence cannot simply cease to be a redundancy.

What changes is:

the selected identity object;

the stabilizer of the vacuum;

the observable spectrum;

and the effective low-energy description.

Thus the phrase gauge selection refers to selection of automorphism structure, not destruction of mathematical equivalence.

After selection, local gauge redundancy remains as the freedom to choose presentations within the selected sector.

The theory should preserve Elitzur-type reasoning in the appropriate lattice context.

A local gauge-dependent field expectation value cannot serve directly as a physical order parameter without gauge fixing.

Sector selection must be diagnosed through gauge-invariant observables.

Possible order parameters include:

free-energy differences;

Wilson-loop behavior;

defect condensation;

gauge-invariant matter composites;

stabilizer classes;

topological observables;

or boundary response.

For finite identity sectors, the global sector label sigma is gauge invariant because different disconnected components are not related by local gauge transformation.

Its probability can therefore serve as an order parameter for identity-sector dominance.

Within one sector, residual symmetry should be diagnosed through gauge-invariant quantities rather than raw psi_v.

The selection mechanism may produce symmetry enhancement as well as reduction.

Suppose a low-temperature sector has a larger automorphism group than competing sectors.

Then cooling increases effective symmetry.

Alternatively, high-energy fluctuations may favor a large-symmetry sector entropically, while low energy selects a smaller group with lower coherence cost.

Both patterns are possible.

The theory should not assume that symmetry always increases with energy or always decreases through cooling.

The result must follow from the calculated free energies.

A further possibility is emergent gauge redundancy.

A microscopic model may have no exact local gauge symmetry, but its low-energy subspace may possess an effective redundancy and gauge field.

In the present framework, this could occur if many microscopic identity configurations collapse into one effective moduli sector whose automorphisms act as low-energy gauge transformations.

This is distinct from selecting one microscopic automorphism group.

It would show that gauge structure itself emerges through coarse-graining.

Such a mechanism may be especially relevant to condensed-matter analogues and quantum error-correcting systems.

The first paper should focus on exact microscopic automorphism selection.

Emergent gauge redundancy under coarse-graining can be a subsequent development.

The selected sector must also support viable dynamics.

A group may dominate the vacuum partition function but fail to produce stable matter, acceptable continuum behavior, or chiral representations.

Selection based on vacuum free energy alone is insufficient for a ToE.

The physically viable sector must satisfy a hierarchy of tests:

dominant or metastable quantum weight;

stable continuum limit;

nontrivial but controlled defect spectrum;

consistent matter representations;

anomaly cancellation;

appropriate low-energy forces;

and compatibility with emergent geometry.

The dynamical-selection program is therefore not a single calculation.

It is a sequence of filters narrowing the candidate identity structures.

9.5 Symmetry Selection as a Non-Inserted Result

A genuinely new result must not be encoded directly into the input.

The strongest first target is a preferred automorphism sector emerging from common rules.

Suppose the candidate set contains groups:

G_1, G_2, ..., G_N.

The theory uses:

the same complex K;

the same boundary conditions;

the same normalized face-cost prescription;

the same temperature or Euclidean extent;

the same structural prior;

and no sector-specific tuning.

Calculate:

Z_i;

F_i;

P_i;

and the sector observables.

A non-inserted selection occurs when one sector i satisfies:

F_i < F_j

for all j != i

throughout a finite parameter region, and when this result is robust under:

lattice enlargement;

small coupling changes;

alternative but reasonable normalization;

and inclusion of quantum stabilizer weights.

The result is stronger if the selected sector also supports:

a stable defect spectrum;

a mass gap;

nontrivial matter representations;

or a favorable continuum scaling trajectory.

A single small-lattice free-energy ordering is not sufficient.

It may be a finite-size artifact.

The selection should be tested on a sequence of complexes.

Let:

Delta F_ij(L)
= F_i(L) - F_j(L).

If:

Delta F_ij(L)

approaches a stable sign and extensive scaling as L increases, the preference is more credible.

If the difference vanishes, the sectors may be degenerate.

If its sign changes repeatedly, no stable selection has been established.

The result should also be compared with null models.

Randomize the multiplication tables while preserving group order where mathematically possible through comparison with non-group quasistructures.

Replace the class function by a flat cost.

Remove plaquette coupling.

Remove automorphism weighting.

Use only group order and no multiplication structure.

These ablations reveal what causes the selection.

If the same group always dominates solely because it has the smallest order, the result may reflect a trivial counting bias.

If selection depends on conjugacy structure, representation content, and holonomy compatibility, it is more physically informative.

A second non-inserted result is a preferred representation sector.

Within a selected group G, include a principled set of matter representations r.

Define:

Z_r

for each representation-coupled sector.

Determine whether one representation or combination minimizes free energy or supports uniquely stable defects.

This would begin to address why matter occupies particular charge representations.

Again, arbitrary representation-specific mass terms must be avoided.

A third result is a universal charge spectrum.

If stable defects can carry only certain representations or holonomy classes, the model may restrict allowed charges.

The restriction should follow from:

topology;

coherence constraints;

fusion consistency;

or anomaly cancellation,

rather than from manually excluding alternatives.

A fourth result is a mass hierarchy.

If several defect sectors possess calculable gaps:

Delta_1, Delta_2, ..., Delta_n,

and the ratios:

Delta_i / Delta_j

are robust, the theory produces a quantitative spectrum.

A fifth result is a preferred effective dimension.

If sector competition is coupled to the relational complex and one sector supports a stable dimension inferred from diffusion or volume growth, the gauge and geometric structures may be selected together.

This is more ambitious and should follow the gauge-only work.

The minimum publishable non-inserted result should be stated before computation.

For example:

Under a universal normalized coherence action on all finite groups up to order N, determine whether the quantum free-energy density selects a proper subset of automorphism sectors in the thermodynamic scaling direction.

This target is objective.

A null result is informative.

The calculation may show that all sectors are equivalent after normalization.

It may show that the smallest group always dominates trivially.

It may show that selection depends sensitively on arbitrary prior choice.

Any of these results would indicate that additional physical principles are required.

The theory should not treat every outcome as confirmation.

A preferred sector is meaningful only when competing explanations have been excluded.

The strongest long-term target is the Standard Model gauge structure.

However, attempting to select:

SU(3) x SU(2) x U(1)

directly from an unrestricted class of compact groups would be premature.

The finite research sequence should be:

select among small finite groups;

identify structural features associated with dominance;

extend the rule to families of Lie groups;

include matter and anomaly constraints;

test product groups and quotient structures;

and only then compare with the Standard Model.

Finite groups may serve as toy models rather than literal approximations of continuous gauge symmetry.

Their purpose is to test whether the selection mechanism works at all.

A successful mechanism should ultimately explain not merely why a group has a certain order or rank, but why its representation theory supports the observed matter and interactions.

The selected object must therefore be evaluated as a complete identity structure, not just by the name of its automorphism group.

The relevant data may include:

the group;

its representations;

its higher extensions;

its allowed defect classes;

its coupling to geometry;

and its anomaly structure.

The ToE target is not only gauge-group selection.

It is selection of an entire consistent physical category.

9.6 Limitations, Falsification, and Research Program

The gauge-selection proposal is one of the most ambitious parts of Univalent Relational Dynamics and therefore requires especially strict limitations.

The first limitation is dependence on the candidate space.

A theory can select only among the sectors it includes.

If the correct physical identity structure is absent, the result is irrelevant.

If the candidate list is designed around the desired answer, selection is circular.

The construction of the candidate space must therefore be independently motivated.

The second limitation is dependence on the prior measure.

The weights omega_i influence sector probabilities.

There is no universally established measure over possible groups, groupoids, or physical theories.

Equal weight per sector is simple but not necessarily fundamental.

Weight by group order, representation dimension, or description complexity introduces different biases.

The result must report this dependence.

A genuine selection principle should be robust across a reasonable class of priors or derive the prior from a higher framework.

The third limitation is normalization across groups.

A Wilson-like action for different groups may not be directly comparable without a common physical scale.

The coupling normalization, representation choice, and maximum face cost affect free energies.

The theory must establish a principled cross-sector normalization.

The fourth limitation is finite-size behavior.

Small complexes may favor certain groups for purely combinatorial reasons.

Thermodynamic and continuum trends must be examined.

The fifth limitation is lack of realistic matter.

Pure gauge selection does not determine the physically observed universe.

Matter representations, chirality, anomaly cancellation, and symmetry breaking can alter the preferred sector.

The sixth limitation is Euclidean interpretation.

A dominant Euclidean partition-function sector does not automatically imply a dynamically selected universe.

A real-time transition mechanism, initial conditions, and causal evolution are required.

The seventh limitation is the distinction between fundamental and effective gauge structure.

A group selected in the finite model may be an effective low-energy symmetry rather than a fundamental automorphism group.

The theory should remain open to both interpretations until scale dependence is analyzed.

The eighth limitation is computational complexity.

The number of configurations grows rapidly with:

group order;

number of edges;

number of object sectors;

and matter-state dimension.

Exact enumeration becomes infeasible quickly.

Monte Carlo or tensor-network methods may be required.

Sector transitions can create severe sampling barriers.

The ninth limitation is continuum consistency.

A finite sector may possess an attractive free energy but no acceptable continuum limit.

Such a sector cannot describe fundamental physics.

The tenth limitation is gravitational coupling.

The selected gauge structure must coexist consistently with the emergent spacetime sector.

A gauge group that dominates on a fixed complex may behave differently when geometry is dynamical.

These limitations define clear falsification criteria.

The simple gauge-selection hypothesis fails if sector dominance is entirely determined by an arbitrary prior.

It fails if reasonable normalizations reverse the result unpredictably.

It fails if no preference survives finite-size scaling.

It fails if the dominant sector lacks stable physical observables or a continuum limit.

It fails if the result merely selects the smallest or largest group through obvious counting.

It fails if arbitrary sector-specific identity costs are required.

It fails if the selected sector changes under a relabeling or equivalent presentation.

It fails if the moduli-groupoid weighting cannot be defined consistently.

It fails as a ToE mechanism if it cannot be extended to realistic continuous groups and matter.

A staged computational program can test these conditions.

Stage 1 compares one-object groupoids BG for all finite groups in a small order range.

Use a common pure face action and identical complexes.

Calculate exact partition functions and stabilizer distributions.

Stage 2 increases lattice size and studies free-energy density.

Stage 3 compares alternative normalization and prior rules.

Stage 4 adds simple matter through a universal representation prescription.

Stage 5 introduces defect sectors and compares mass gaps.

Stage 6 allows spatial coexistence of identity sectors through domain-wall data.

Stage 7 studies real-time or transfer-matrix transitions between sectors.

Stage 8 extends structural insights to compact Lie groups.

Stage 9 imposes anomaly and continuum-consistency filters.

Stage 10 investigates whether product groups and symmetry-breaking chains resembling observed physics are selected.

GenesisBoard can support part of this program if extended to generate:

candidate identity objects;

automorphism groups;

transport networks;

face holonomies;

sector partitions;

and coherence-cost statistics.

Its existing emphasis on categorical composition and coherence makes it a natural exploratory laboratory.

However, exact group-theoretic benchmark models should be developed independently so that simulation results can be checked against known finite calculations.

The main output of the first gauge-selection paper should not be a claim that the Standard Model has been derived.

It should be one of three honest outcomes.

The first possible outcome is positive selection.

A restricted class of automorphism sectors dominates robustly under common rules.

The second is conditional selection.

Dominance occurs only after a particular structural prior or matter coupling is added.

This identifies the missing principle required by the theory.

The third is no selection.

The sectors remain degenerate or their ordering is arbitrary.

This would show that coherence cost alone cannot explain the gauge group.

All three outcomes advance the program because they constrain what a viable equivalence-first ToE must contain.

The central conclusion of this section is that gauge structure can be made into a dynamical question.

A fixed gauge group is the automorphism group of one already selected identity object.

A heterogeneous candidate theory contains several identity sectors.

The coherence action and moduli-groupoid measure assign them quantum weights.

A dominant sector defines an effective gauge group.

Matter condensation may then reduce that group to a residual symmetry.

The proposal becomes explanatory only when the selected sector is not encoded in the inputs and when its dominance survives consistency, scaling, and normalization tests.

The next section investigates whether the same transport structure can generate relational distance, effective dimension, curvature, and a bridge toward gravitational dynamics.

10. Emergent Geometry and the Gravity Bridge

10.1 Relational Distance

The finite oriented 2-complex used in the preceding sections was introduced as a compositional scaffold rather than as a discretization of a preexisting physical space. Vertices represent local identity contexts, edges represent admissible transport, and faces test the coherence of transport around loops. The complex specifies which relations are available, but it does not initially assign physical distances, angles, durations, or metric geometry.

A foundational theory should not simply place the equivalence field on a fixed spacetime and then claim that spacetime has emerged. It must identify conditions under which metric and geometric structure can be reconstructed from the transport dynamics itself.

The first step is to define relational proximity.

Two local contexts should be regarded as physically near when one can be coherently identified with the other at low cost.

Let v and w be vertices of the complex K. Let:

p : v -> w

denote an admissible oriented path:

p = (e_1,e_2,...,e_n).

Each edge carries an identity transport:

U_e : A_s(e) -> A_t(e).

Assign a nonnegative local transport cost:

c_e = C_e(A_s(e),A_t(e),U_e,Phi),

where Phi denotes the complete configuration and C_e may depend on the local identity objects, the transport morphism, neighboring fields, and the coherence environment.

The cost of the path may be defined additively:

C(p) = sum over e in p of c_e.

The candidate relational distance is then:

d_Phi(v,w)
= inf over admissible paths p:v->w of C(p).

The subscript Phi emphasizes that the distance depends on the physical equivalence configuration. Geometry is therefore not fixed independently of the fields. A change in transport, identity type, matter content, or defect structure may alter the distance.

This definition expresses the central proposal:

distance is the least cost of coherently transporting physical identity between two contexts.

The cost is not merely the number of edges crossed. A path containing many low-cost equivalences may be physically shorter than a path containing fewer high-cost transformations.

The combinatorial adjacency of the complex supplies possible routes. The physical transport cost determines their effective length.

The simplest choice assigns one positive cost to each edge:

c_e = l_e.

Then d_Phi reduces to the ordinary weighted shortest-path metric.

This construction is mathematically familiar, but it does not yet derive geometry from equivalence. It merely assigns lengths to edges.

A stronger construction derives c_e from the relational fields.

For example, one may define:

c_e
= c_0

- alpha ||D_e psi||^2
- gamma Q_e(U_e)
- delta I_e(A_s(e),A_t(e)).

Here:

c_0

is a baseline transport cost,

||D_e psi||^2

measures matter mismatch,

Q_e(U_e)

measures transport complexity or departure from a locally preferred identification, and:

I_e

measures identity-sector incompatibility.

Another possibility defines transport cost through the local action response.

Let Phi_e(lambda) be a deformation that transports a test identity across edge e. Then:

c_e
= minimum excess action required to perform the transport.

In this interpretation, distance is derived from the energetic cost of comparison rather than inserted as an independent edge label.

A still stronger formulation uses transition amplitudes.

If the quantum amplitude for coherent transport between nearby contexts is:

A_e,

one may define an effective cost:

c_e
= -ell_0 log |A_e|,

where ell_0 is a conversion scale.

Strongly connected contexts with large transport amplitude are near.

Weakly connected contexts with exponentially suppressed transport are far.

The distance may then be inferred from the decay of gauge-invariant correlation functions.

Suppose:

C(v,w)

is a relational two-point function. In a massive phase:

C(v,w)
approximately proportional to
exp[-d(v,w)/xi].

Then d can be reconstructed, up to scale and corrections, from:

d(v,w)
approximately equals
-xi log[C(v,w)/C_0].

This provides an operational rather than purely formal definition of distance.

The theory should distinguish several candidate geometries.

Combinatorial geometry depends only on graph adjacency.

Action geometry depends on local coherence costs.

Correlation geometry depends on decay of physical observables.

Information geometry depends on distinguishability or mutual information.

Causal geometry depends on which directed processes can occur.

A successful emergent geometry should show that these constructions become compatible in an appropriate regime.

If shortest-path cost, correlation decay, diffusion, and causal propagation all define unrelated structures, no unique physical spacetime has emerged.

The relational distance should satisfy gauge invariance.

If Phi and Phi' are related by a local change of presentation, then:

d_Phi(v,w) = d_Phi'(v,w).

This requires each edge cost or complete path cost to be defined from gauge-invariant quantities.

A cost depending directly on a raw link variable U_e is generally gauge dependent.

A valid cost may depend instead on:

the norm of a covariant difference;

a class function of relative transport;

the minimum action over gauge-equivalent representatives;

a transported inner product;

or another descending construction.

One may define an explicitly invariant edge cost by minimization:

c_e([Phi])
= inf over g_s,g_t
C_raw(g_t U_e g_s^-1,

rho(g_s)psi_s,
rho(g_t)psi_t).

This removes cost caused solely by a poor choice of endpoint frame.

However, independent minimization on every edge may destroy global consistency. The optimal gauge choices for neighboring edges may be incompatible.

A more faithful distance may minimize the total path cost over both the path and a coherent choice of presentations along it.

Let g assign a local frame to every vertex of p. Then:

d(v,w)
= inf over p and g
C(p,g).

This construction resembles gauge fixing along a path while respecting composition.

In many models, the covariant quantities make explicit minimization unnecessary.

The distinction between physical distance and identity transport cost should also be examined.

A transformation can be costly because of internal gauge rotation without implying large spatial separation.

Conversely, two spatially distant contexts might be connected by a low-action topological channel.

The theory must identify conditions under which coherence cost behaves as geometric distance rather than as an unrelated internal complexity measure.

One possible condition is universality.

If all sufficiently low-energy test identities experience the same effective distance, the geometry is universal.

If different representation types infer different incompatible distances, the structure resembles multiple effective metrics or species-dependent propagation.

A gravitational geometry should satisfy an equivalence-like principle: suitably localized test structures should respond to one common relational metric, at least in the appropriate regime.

The finite model can test universality by comparing distances inferred from several probe representations.

Let:

d_r(v,w)

be the distance inferred from representation r.

A universal geometric phase requires:

d_r(v,w)
approximately equals
z_r d(v,w)

with at most a representation-dependent conversion scale z_r, or preferably:

d_r(v,w)
approximately equals
d(v,w)

after common normalization.

Large irreducible differences would indicate that no unique spacetime metric has emerged.

The baseline constant c_0 also requires explanation.

If c_0 > 0, every edge traversal has a minimum cost, and graph distance contributes directly.

If c_0 = 0, a chain of perfectly coherent equivalences may have zero total length, collapsing distinct vertices into one physical location.

Such collapse may be appropriate if the vertices are merely redundant presentations.

The physical geometry should therefore be defined on equivalence classes of contexts rather than on raw vertices when zero-cost transport identifies them.

Define:

v ~_0 w

when:

d(v,w) = 0

and:

d(w,v) = 0

in a directed model.

The effective point set is then the quotient by zero-distance equivalence.

This follows the same equivalence-first logic as gauge reduction.

Multiple vertices may be distinct computational presentations of one physical location.

Geometry begins only after zero-cost redundancy is removed.

Relational distance therefore performs two tasks:

it identifies which contexts are physically indistinguishable;

and it assigns separation among the remaining effective locations.

This construction is the first bridge from identity transport to geometry.

10.2 Conditions for a Metric

The shortest-cost construction does not automatically produce an ordinary metric.

A metric d on a set X must satisfy:

d(x,y) >= 0;

d(x,y) = 0 if and only if x = y;

d(x,y) = d(y,x);

d(x,z) <= d(x,y) + d(y,z).

These are positivity, separation, symmetry, and the triangle inequality.

Each property imposes a physical condition on the relational theory.

Positivity is satisfied when every allowed transport cost is nonnegative.

A Euclidean coherence cost naturally has this property.

In a Lorentzian or phase-based theory, the fundamental action need not be nonnegative. A positive spatial or informational cost may have to be extracted from a different quantity, such as absolute amplitude, Euclidean continuation, or optimal control cost.

Separation is more subtle.

If:

d(v,w) = 0

for distinct vertices v and w, then d is a pseudometric rather than a metric.

This need not be a failure.

The vertices may represent distinct presentations connected by costless physical equivalence.

The correct metric space is obtained after quotienting by the zero-distance relation.

Let:

X_eff = V / ~_0.

Then define:

d_eff([v],[w]) = d(v,w).

This is well defined only if zero-distance representatives produce the same distances to all other classes.

The theory must verify this consistency.

Symmetry requires:

d(v,w) = d(w,v).

If each edge transport is reversible and satisfies:

c(e^-1) = c(e),

then reversing a path preserves its cost, and symmetry follows.

However, the complete physical theory contains noninvertible processes.

Causal propagation, dissipation, measurement, decay, and time-directed composition may produce:

d_forward(v,w) != d_forward(w,v).

The appropriate structure may then be a directed metric, quasi-metric, causal order, or Finsler-like geometry rather than a Riemannian metric.

The theory should not impose symmetry if the underlying processes are physically directed.

A possible separation is:

reversible equivalence transport generates spatial distance;

noninvertible process composition generates temporal or causal order.

This suggests that spatial geometry may arise from the symmetric part of the transport structure, while time emerges from directed morphisms.

Define:

d_sym(v,w)
= [d_forward(v,w) + d_forward(w,v)] / 2,

and an asymmetry:

a(v,w)
= d_forward(v,w) - d_forward(w,v).

The symmetric component may approximate spatial separation.

The antisymmetric or directed component may encode causal orientation or thermodynamic arrow.

This proposal requires substantial development, but the finite model can measure whether such a decomposition becomes stable.

The triangle inequality follows naturally when path cost is additive.

If p connects x to y and q connects y to z, their concatenation connects x to z with:

C(q o p) = C(q) + C(p).

Since d(x,z) is the infimum over all x-to-z paths:

d(x,z)
<= d(x,y) + d(y,z).

If path costs include nonadditive discounts, interference, memory, or global constraints, the triangle inequality may fail.

Such failure does not automatically invalidate the relational geometry.

It may indicate:

nonlocal transport;

collective shortcuts;

context-dependent distance;

quantum superposition of routes;

or a nonmetric effective structure.

The theory must determine which regime reproduces ordinary geometry.

A continuum spacetime requires more than a metric on a finite vertex set.

It requires local regularity, dimensionality, approximate manifold structure, and a notion of smooth variation.

The first finite criterion is local consistency.

For nearby vertices, distances should vary gradually under small changes of the fields.

A single edge perturbation should not arbitrarily reorganize the entire geometry unless the system is at a critical transition or contains nonlocal constraints.

The second criterion is extensivity.

For long homogeneous chains, distance should grow approximately linearly with repeated transport:

d(v_0,v_n)
approximately equals
n ell_eff

for sufficiently large n and fixed local conditions.

Strong saturation or superlinear growth may correspond to compactification, confinement, fractal geometry, or other phases.

The third criterion is approximate isotropy.

In a candidate homogeneous phase, the distance should not depend strongly on arbitrary lattice directions after coarse-graining.

A square lattice is anisotropic microscopically but may approach rotational invariance at long scale.

The relational cost must be tested for similar restoration.

The fourth criterion is universality across refinements.

Suppose the complex K is subdivided into K'.

The effective distance between corresponding macroscopic regions should remain stable after coupling rescaling.

If subdivision doubles every distance without an interpretable scale change, the geometry is only combinatorial.

The fifth criterion is compatibility with physical propagation.

Wave packets, defects, or correlations should propagate according to the emergent distance.

If the shortest-cost metric predicts one light cone while actual excitations propagate according to another, the metric lacks operational significance.

The sixth criterion is local reconstructibility.

Around a generic effective point, the distance relations among nearby points should permit an approximate tangent structure or local coordinate chart.

One may test whether the local distance matrix is approximately embeddable in R^d, Minkowski space, or another homogeneous model.

Methods include:

multidimensional scaling;

local Gram-matrix analysis;

spectral embedding;

and comparison of small geodesic triangles.

The seventh criterion is geodesic stability.

Minimum-cost paths should vary continuously under small changes of endpoints in a smooth regime.

Frequent discontinuous switching among unrelated routes may signal a highly discrete, glassy, or multiply connected geometry.

The eighth criterion is compatibility with curvature measures.

Distances around neighboring regions should correlate with holonomy-derived curvature.

A model in which the metric and connection structures are independent has not yet unified geometry.

The finite theory should test whether a relation analogous to metric compatibility emerges.

A path transport may act on internal identity while d is derived from its cost.

For a gravitational interpretation, the transport should also determine how local frames, directions, or causal cones compare.

This requires richer local identity objects than simple internal group elements.

One may assign a frame-like structure to each A_v and let automorphisms include rotations, boosts, or generalized local linear transformations.

Then edge transports compare frames, and face holonomy yields geometric curvature.

In the initial model, the metric construction remains more abstract.

It establishes relational separation but not yet a full tangent geometry.

The result may be called pregeometry.

Pregeometry consists of:

effective points;

relational distances;

neighborhoods;

path composition;

and curvature-like loop obstruction.

A manifold description is a possible continuum phase of this structure, not an initial assumption.

The theory should therefore use cautious terminology.

A valid finite metric is not yet spacetime.

A continuum manifold is not yet Lorentzian spacetime.

A Lorentzian metric is not yet general relativity.

Each level requires additional recovery conditions.

10.3 Effective Dimension

Dimension should not be inserted merely by choosing a square, cubic, or four-dimensional lattice.

If the complex serves only as a regulator for a known theory, its dimension is an external input.

If the model aims to explain spacetime, effective dimension must be inferred from relational behavior.

Several definitions of dimension are available in discrete and irregular systems.

They need not agree at small scales.

A geometric phase is characterized partly by the convergence of these definitions over an intermediate or large-scale regime.

The first measure is volume-growth dimension.

Choose an effective point v and define the ball:

B(v,R)
= {w | d(v,w) <= R}.

Let:

N_v(R)
= |B(v,R)|

or an automorphism-corrected measure of the ball.

If:

N_v(R)
approximately proportional to
R^d

over a range of scales, define:

d_vol
= d log N_v(R) / d log R.

For a regular d-dimensional Euclidean lattice, this approaches d away from finite-size and lattice-scale effects.

In a relational geometry, d_vol may vary with R.

This scale dependence can reveal dimensional flow.

The system may appear high dimensional at short scale and lower dimensional at long scale, or the reverse.

The second measure is spectral dimension.

Define a diffusion process on the effective weighted graph or relational complex.

Let:

P_return(t)

be the probability that a random walker returns to its starting point after diffusion time t.

For a d-dimensional homogeneous Euclidean space:

P_return(t)
approximately proportional to
t^(-d/2).

Define:

d_spec(t)
= -2 d log P_return(t) / d log t.

The spectral dimension probes propagation rather than merely counting vertices.

It is therefore sensitive to:

edge weights;

bottlenecks;

shortcuts;

trapping;

and coherence-induced transport suppression.

The third measure is Hausdorff-like dimension.

Using the relational metric, cover the effective space with balls of radius epsilon and study how the minimum number of balls scales with epsilon.

This is more difficult on finite systems but useful in fractal regimes.

The fourth measure is embedding dimension.

Given a distance matrix among a local set of points, determine the smallest d for which the distances can be approximately embedded in a standard d-dimensional geometry.

This may be tested through the rank and signature of a reconstructed Gram matrix.

The fifth measure is information dimension.

If probability weight is distributed nonuniformly across physical configurations or effective points, dimension may be inferred from the scaling of entropy with resolution.

The sixth measure is categorical or compositional dimension.

One may count the growth of independent paths, commuting diagrams, or higher cells as a function of relational radius.

GenesisBoard-like observables may be particularly useful here.

For example, the number of distinct morphism paths reaching an equivalence class may scale differently in low- and high-dimensional phases.

The seventh measure is causal dimension.

If directed processes define causal intervals, the number of elements in an interval may scale with proper time in a manner characteristic of Lorentzian dimension.

This requires a directed extension beyond the present reversible metric model.

The theory should not choose one definition solely because it yields the desired number.

A convincing emergent dimension requires agreement among several operational measures.

For example:

d_vol
approximately equals
d_spec
approximately equals
d_embed

over a stable scale range.

Disagreement may indicate:

fractal structure;

anisotropy;

multiscale geometry;

nonlocality;

or absence of a manifold phase.

The underlying combinatorial dimension of K may differ from the effective physical dimension.

A high-connectivity graph can have large or infinite volume-growth dimension even if drawn in two dimensions.

A sparse network embedded in a high-dimensional lattice may behave effectively one dimensional.

The relational costs may suppress most edges, producing a lower-dimensional low-cost backbone.

Conversely, long-range low-cost equivalences may increase effective dimension.

This is why the physical dimension must be measured from d_Phi or propagation rather than inferred from the diagram used to represent the complex.

A dynamic model may select dimension through free-energy competition.

Consider families of complexes or relational phases with different effective dimensions.

The partition function includes:

action cost;

configuration entropy;

defect stability;

and transport efficiency.

A dimension may be favored because it provides the best balance between local coherence and global connectivity.

Too low a dimension may prevent sufficient relational complexity or stable defect linking.

Too high a dimension may produce excessive connectivity, dilute topological protection, or increase coherence cost.

The preferred dimension could therefore emerge as an optimum.

This idea should be treated as a hypothesis.

To test it, the candidate ensemble must permit different effective dimensions under common microscopic rules.

If the theory begins with a fixed four-dimensional hypercubic lattice, it cannot claim to derive four dimensions.

Possible ensembles include:

random graphs;

dynamical triangulations;

cell complexes of varying connectivity;

GenesisBoard-generated categories;

or adaptive networks whose edges are created and removed according to coherence cost.

For each configuration, calculate:

effective dimension;

action;

entropy;

defect spectrum;

and partition weight.

Then determine which dimensional regime dominates.

The theory should also distinguish spatial dimension from spacetime dimension.

The finite Euclidean complex may possess d total directions.

A Lorentzian theory requires one temporal direction with different signature or causal role.

Recovering an effective dimension near four is insufficient if all four directions remain Euclidean.

The emergence of 3+1 structure requires:

three approximately symmetric spatial directions;

one directed or Lorentzian temporal dimension;

local causal cones;

and an effective finite propagation speed.

Several mechanisms might distinguish time.

One possibility is that invertible equivalence relations generate spatial adjacency while noninvertible processes generate temporal order.

Another is that the coherence action undergoes spontaneous signature selection.

Another is that one direction is selected by boundary conditions or a thermodynamic arrow.

Another is that a transfer-matrix construction identifies one compositional direction as time.

The current finite gauge model uses Euclidean time as a calculational input for quantization.

That is not yet an emergence of physical time.

The paper should state this limitation clearly.

A meaningful dimensional result should also survive matter coupling.

The effective dimension seen by gauge probes, scalar defects, and diffusion should agree approximately.

If different fields see different dimensions, the system may exhibit dimensional reduction by sector, which could be interesting but is not ordinary spacetime.

Dimensional flow itself may be a prediction.

Many quantum-gravity approaches find scale-dependent spectral dimension.

Univalent Relational Dynamics may generate similar behavior if short-scale coherence paths differ from long-scale effective transport.

The relevant novelty would not be dimensional flow by itself.

It would be a derivation connecting that flow specifically to the cost, multiplicity, and obstruction of equivalence transport.

The first computational target should be modest.

Given a family of finite relational complexes with dynamically weighted edges, determine whether:

volume-growth dimension;

spectral dimension;

and correlation dimension

converge toward a common value.

Then test how that value depends on:

coherence coupling;

defect density;

identity-sector choice;

and system size.

A non-inserted preferred dimension would be a major result.

A dimension merely inherited from the chosen lattice would not be.

10.4 Curvature Comparison

The model contains at least two distinct notions related to curvature.

The first is holonomy curvature.

For a face f:

H_f != id

indicates failed identity closure.

The second is metric curvature.

If relational distances define an effective geometry, the deviation of geodesic balls, triangles, or volume growth from flat-space behavior indicates curvature.

A gravity bridge requires these notions to become related.

In ordinary differential geometry, a connection may be independent of the metric.

In general relativity, the Levi-Civita connection is determined by the metric through torsion-free metric compatibility.

Its holonomy reflects spacetime curvature.

In gauge theory, the internal connection generally describes a different bundle and is not determined by the spacetime metric.

The present framework initially treats identity transport abstractly.

To claim emergent gravity, it must identify a sector in which the connection governing physical frame transport is compatible with the emergent metric.

The finite model can compare several curvature measures.

The face coherence curvature is:

kappa_f
= K(H_f).

This is gauge invariant but depends on the selected class function.

A small-holonomy Lie-group model may define a Lie-algebra curvature magnitude:

||F_f||^2
approximately proportional to
K(H_f) / area_f^2.

The area_f itself must be derived or assigned.

A metric triangle measure compares the angles or side lengths of local triangles with Euclidean expectations.

For a geodesic triangle with side lengths a,b,c, one may reconstruct approximate angles and calculate an angular excess or deficit.

In a Regge-like simplicial geometry, curvature is concentrated on codimension-two hinges and measured by deficit angles.

If the relational metric is represented by edge lengths on a triangulation, Regge curvature provides a direct discrete comparison.

Graph curvature supplies another family of measures.

Ollivier-type curvature compares the transport distance between probability distributions around neighboring vertices.

Forman-type curvature uses weighted cell incidence.

These graph curvatures are mathematically different from gauge holonomy.

Their agreement should not be assumed.

A successful geometric phase may show correlation between:

holonomy cost;

Regge deficit;

graph curvature;

and geodesic deviation.

A finite geodesic-deviation test is especially relevant.

Take two initially neighboring minimum-cost paths starting with similar directions.

Measure whether their separation grows, shrinks, or remains constant.

This behavior defines an effective tidal curvature.

If regions with nontrivial frame holonomy produce corresponding geodesic deviation, the transport and metric structures are coupled.

The theory should define a local frame object.

Let A_v contain not only an internal identity label but a space of local directions or probes.

Its automorphism group may include a rotation or Lorentz-like group.

Edge transport then maps local frames between vertices.

The holonomy around a face acts on directions.

The relational distance determines geodesics.

Metric compatibility requires that transport preserve the local inner product:

<rho(U_e)x,rho(U_e)y>_(t(e))

<x,y>_(s(e)).

In a Riemannian sector, the automorphisms preserving the metric belong locally to an orthogonal group.

In a Lorentzian sector, they belong to a Lorentz group.

If transport preserves the emergent metric and has negligible torsion in the relevant limit, its holonomy can be interpreted as spacetime curvature.

This would be a substantial extension beyond internal gauge theory.

The local identity object must then encode tangent-frame structure or a categorical analogue.

One route uses a frame bundle.

The identity object A_v includes a local frame.

Aut(A_v) contains frame transformations.

The transport field is a discrete connection.

An additional soldering or coframe field relates internal frame directions to displacements in the relational complex.

Without such a relation, frame rotation remains internal and does not define spacetime geometry.

The finite analogue may assign to each oriented edge a local displacement vector:

E_e in V_(A_s(e)),

transforming covariantly under frame automorphisms.

The metric length is:

l_e^2
= <E_e,E_e>.

The connection transports E_e between frames.

Torsion measures failure of edge displacements to close around a face after transport.

Curvature measures failure of frame transport to close.

This produces a discrete gauge-gravity structure closer to Cartan geometry.

The present paper need not complete this construction, but it should identify it as the likely bridge from abstract identity transport to gravitational geometry.

The equivalence-first dictionary becomes:

local frame = presentation of spatial or spacetime identity;

connection = rule for comparing frames;

coframe or soldering data = relation between frame identity and physical displacement;

torsion = failure of transported displacements to close;

curvature = failure of transported frames to return trivially;

metric = invariant comparison structure on frames;

gravity = dynamics of the coupled metric-transport system.

A purely holonomy-based internal gauge model cannot by itself derive all of this.

The model must add the structure connecting automorphisms to geometry.

The relational cost may help determine the coframe.

For each edge, the least cost of identity transport defines a length.

The direction associated with that transport defines an effective displacement.

Local collections of such displacements may reconstruct a frame.

If the reconstruction is consistent across overlaps, a metric geometry emerges.

If it fails, the obstruction may appear as torsion or nonmetricity.

The theory can therefore accommodate more general geometries before restricting to general relativity.

Possible phases include:

Riemannian geometry;

Lorentzian geometry;

Riemann-Cartan geometry with torsion;

Weyl-like geometry with scale transport;

Finsler geometry with direction-dependent cost;

or nonmetric relational geometry.

General relativity corresponds to a specific low-energy phase.

The theory must derive or justify why that phase dominates.

The curvature comparison should proceed in stages.

First, calculate holonomy curvature on a fixed weighted complex.

Second, derive shortest-cost distances.

Third, compute graph or Regge curvature.

Fourth, test statistical correlation between the measures.

Fifth, introduce frame-valued identity objects and metric-compatible transport.

Sixth, study refinement toward a continuum connection.

Seventh, compare the effective action with Einstein-Cartan or Einstein-Hilbert dynamics.

The first four stages are feasible in a finite computational model.

The later stages require substantial formal development.

A successful finite result would show that regions of high identity-closure cost systematically distort relational distances and geodesic flow.

That would provide a concrete beginning for the claim that geometry records coherence obstruction.

A stronger result would derive the metric curvature entirely from the same variables that define holonomy.

A still stronger result would produce the correct gravitational field equations.

The theory should maintain the distinction among these achievements.

10.5 Entropy of Equivalence Fibers

The equivalence-first framework proposes that entropy can be understood as the multiplicity of presentations realizing one coarse physical meaning.

Let:

q : P -> M

map a presentation space P to a space or groupoid M of physical meanings.

For a coarse physical state m in M, the fiber:

q^-1(m)

contains the microscopic or presentation-level configurations associated with m.

The simplest entropy is:

S_eq(m)
= k_B log N(m),

where N(m) is the number of distinguishable microscopic presentations in the fiber.

This resembles Boltzmann entropy.

The distinctive element is that the fiber is not assumed to be an unstructured set.

Its presentations may possess:

gauge equivalences;

automorphisms;

topology;

higher paths;

and different physical weights.

A symmetry-corrected finite multiplicity is given by the homotopy cardinality:

Vol_h(q^-1(m))
= sum over [x] in q^-1(m)
1 / |Aut(x)|.

Then define:

S_eq(m)
= k_B log Vol_h(q^-1(m)),

when this quantity is positive and the statistical interpretation is appropriate.

More generally, configurations carry action weights.

The restricted partition function for macrostate m is:

Z_m
= sum over [Phi] with q(Phi)=m
exp[-S_URD(Phi)/hbar]
/ |Aut(Phi)|.

The thermodynamic entropy may be derived from the free energy rather than from unweighted counting.

At temperature T:

$F_m = -k_B T \log Z_m$.

Then:

$S_m = -\partial F_m / \partial T$.

The unweighted equivalence-fiber entropy is recovered in a regime where all allowed microconfigurations in the macrofiber have equal energy.

The theory should distinguish:

presentation multiplicity;

gauge redundancy;

physical microstate multiplicity;

and quantum amplitude multiplicity.

Gauge-related presentations should not be counted as distinct physical microstates.

Physically inequivalent microconfigurations corresponding to one macrostate should be counted.

Automorphism weighting corrects residual self-symmetry.

Coherent quantum alternatives should not always be treated as classical multiplicities.

Their phases may interfere.

Entropy becomes classical only after the relevant alternatives are decohered or coarse-grained.

These distinctions are necessary to avoid overcounting equivalence.

The phrase entropy as multiplicity of equivalent presentation can otherwise be misleading.

If two configurations are strictly gauge equivalent, they represent one physical microstate, not two contributions to thermodynamic entropy.

The entropy arises from multiple physical microstates or presentation structures that become indistinguishable under the chosen coarse observation.

Thus the relevant equivalence is macroequivalence, not necessarily fundamental gauge identity.

Let:

Phi_1 ~_macro Phi_2

when the macroscopic observable map assigns the same outcome:

q(Phi_1) = q(Phi_2) = m.

The configurations may remain physically distinct microscopically.

The fiber contains all such microstates modulo fundamental gauge redundancy.

This gives the corrected structure:

microscopic presentations
-> quotient by gauge equivalence
-> physical microstates
-> coarse-graining map
-> macrostate m.

The entropy counts the physical microstates in the coarse fiber, with symmetry correction where appropriate.

The same framework can describe information loss.

A truncation or coarse-graining map:

tau : R -> R_coarse

sends several higher relational structures to one lower-resolution image.

The entropy increase associated with tau reflects the volume of the preimage:

tau^-1(r_coarse).

If repeated physical processes enlarge the accessible coarse fibers, the thermodynamic arrow may be described as growth of equivalence-fiber volume.

The second law would then state, under appropriate macroscopic conditions:

dS_eq/dt >= 0.

This inequality does not follow from counting alone.

The microscopic dynamics may be reversible.

An increase in coarse entropy requires:

special initial conditions;

coarse-graining;

typicality;

environmental entanglement;

or effective loss of accessible correlations.

The equivalence-first language reorganizes the microstates but does not eliminate the standard statistical-mechanical problem of deriving irreversibility.

The entropy construction becomes relevant to gravity because horizon thermodynamics associates entropy with causal boundaries and inaccessible information.

A horizon divides relational data into accessible and inaccessible regions.

The observer's coarse physical state identifies many underlying configurations behind the boundary.

The corresponding equivalence fiber may possess an entropy proportional to the number or measure of hidden relational presentations.

The gravitational bridge requires much more than this qualitative statement.

One must establish:

which moduli object is being counted;

which boundary data are fixed;

why the entropy scales with area rather than volume;

how automorphism and gauge constraints alter the count;

how the effective temperature arises;

and how entropy variation couples to energy flux.

The finite model can begin by studying subregion entropy.

Partition the complex into a region R and its complement.

Fix gauge-invariant boundary data on the interface.

Count or sum the configurations in the hidden region compatible with those boundary data:

Z_R(boundary data).

Define an effective boundary entropy:

$$S_R = k_B \log Z_R$$

in an equal-weight model, or derive it thermodynamically from the restricted partition function.

Then examine how S_R scales with:

the number of interior vertices;

the number of boundary edges or faces;

the coherence coupling;

and defect content.

In a local gapped gauge theory, physical correlations may cause the leading entanglement or boundary-sensitive contribution to scale with boundary area.

Topological terms may add constant corrections.

The finite groupoid structure may modify these counts through stabilizers and boundary gauge modes.

This provides a direct computational route toward testing an area law for equivalence entropy.

The boundary treatment is crucial.

Gauge theories do not factorize naively into independent Hilbert spaces for spatial subregions because constraints link the boundary.

Edge modes or extended Hilbert-space constructions may be required.

The groupoid formulation may clarify this issue by making boundary automorphisms explicit.

Some transformations that are gauge redundancies in the bulk become physical boundary symmetries.

Their state multiplicity may contribute to entropy.

The theory must decide which boundary transformations are quotiented and which are retained.

A horizon-like entropy may therefore arise partly from boundary relational degrees of freedom.

This is compatible with several existing approaches, but the present framework must provide its own precise finite construction.

The equivalence entropy can also be connected to defect matter.

A defect sector may have an internal degeneracy:

Omega_d.

Its entropy is:

S_d = k_B log Omega_d,

after gauge correction.

Composite defects may possess many relational arrangements corresponding to one coarse particle state.

The automorphism structure may reduce or organize this degeneracy.

Mass and entropy then compete in the free energy:

F_d = E_d - T S_d.

A heavier but highly degenerate defect sector may dominate at high temperature.

This may influence early-universe phase selection.

Again, such interpretation requires real dynamics and should follow the finite statistical calculation.

The main contribution of the equivalence-fiber formulation is to provide one common language for:

statistical multiplicity;

gauge quotienting;

coarse-graining;

measurement truncation;

boundary information;

and sector entropy.

Its success depends on precise counting.

A vague claim that “more equivalent descriptions means more entropy” is insufficient and sometimes incorrect.

The correct question is:

How many physically inequivalent microscopic relational structures are mapped to the same accessible coarse meaning, after gauge redundancy and automorphism symmetry are treated properly?

That number or weighted volume defines the relevant entropy.

10.6 Thermodynamic Gravity Conjecture

A possible bridge from equivalence dynamics to gravitational field equations proceeds through thermodynamics.

The general strategy begins from the observation that gravitational dynamics can be related, under specific assumptions, to horizon entropy, local acceleration temperature, energy flux, and focusing of null congruences.

Univalent Relational Dynamics proposes that the entropy in this relation may arise from equivalence-fiber multiplicity.

The conjectural chain is:

equivalence structure
-> coarse or inaccessible relational multiplicity
-> horizon entropy
-> thermodynamic response
-> geometric focusing
-> gravitational field equation.

Each arrow requires a derivation.

Let a local causal boundary separate accessible and inaccessible relational data for an observer.

Let:

S_eq

denote the entropy associated with physical microconfigurations compatible with the same accessible boundary state.

Suppose an energy or coherence flux:

delta Q

crosses the boundary.

Suppose the observer assigns an effective temperature T to the boundary.

The local thermodynamic relation is:

delta Q = T delta S_eq.

If this relation holds for all sufficiently local causal boundaries, and if the entropy variation is proportional to an appropriate geometric area variation, one may recover an Einstein-like equation of state under the standard geometric assumptions.

The central question is whether URD can derive the ingredients rather than merely borrowing them.

The first required ingredient is local causal structure.

The current Euclidean finite model does not yet contain genuine local Rindler horizons or null congruences.

A Lorentzian or directed extension is required.

The theory must define:

causal accessibility;

local accelerated observers;

null or approximately null boundaries;

and energy flux across those boundaries.

The second required ingredient is temperature.

An acceleration-dependent temperature cannot be assumed without explanation if spacetime and quantum fields are emergent.

The theory must show that restricted observation of the relational vacuum yields a thermal state with:

T proportional to acceleration,

or an analogous finite-model relation.

A transfer-matrix or modular-theoretic construction may be relevant.

The third required ingredient is entropy-area proportionality.

The finite equivalence entropy must satisfy:

delta S_eq
= eta delta A

in the local horizon regime, where eta is a universal coefficient.

If entropy scales instead with volume or depends strongly on probe species, the standard gravitational derivation does not follow.

The fourth required ingredient is a geometric area A.

Area must be defined from the emergent metric or boundary-cell measure.

If geometry is derived from transport cost, the area must be reconstructed consistently from that same relational structure.

The fifth required ingredient is energy flux.

The stress-energy quantity crossing the boundary must be derived from variations of the coherence action or quantum expectation values.

It should satisfy a conservation law.

The sixth required ingredient is focusing.

A discrete or continuum analogue of the Raychaudhuri equation must relate the change in cross-sectional area to curvature along the congruence.

The seventh required ingredient is universality.

The resulting relation must hold for arbitrary local causal boundaries and matter fluxes in the appropriate limit.

Only then can it constrain the geometric field equation.

The conjectural continuum result is:

G_mu_nu + Lambda g_mu_nu
= 8 pi G_N T_mu_nu.

In URD language:

the left side describes the macroscopic geometry of identity transport;

the right side describes the local flux and concentration of coherence cost or matter energy;

the cosmological term reflects a vacuum contribution or integration constant.

This interpretation is attractive but does not replace the derivation.

The theory must recover:

the tensor form;

the numerical coupling;

the conservation condition;

and the low-energy limit.

A finite model can test precursors of the thermodynamic relation.

Choose a subregion R with a boundary.

Define:

S_eq(R)

from the restricted moduli-groupoid partition function.

Introduce a controlled change that transports energy or coherence across the boundary.

Measure:

delta E_R;

delta S_eq(R);

and the change in effective boundary geometry.

Test whether a relation of the form:

delta E_R
approximately equals
T_eff delta S_eq(R)

holds over a regime with a well-defined effective temperature.

This is not yet a gravitational field equation.

It is a finite thermodynamic consistency test.

A stronger test varies the boundary and compares entropy change with relational area change:

delta S_eq
approximately equals
eta delta A_eff.

A still stronger test measures focusing of low-cost paths or causal trajectories near a concentration of defect energy.

If added defect energy changes the relational metric so that neighboring geodesics focus, the model exhibits a primitive gravitational response.

The full bridge would connect all three:

energy flux;

entropy change;

and geometric focusing.

The equivalence entropy may also clarify why gravity is universal.

If geometry is determined by the total relational cost and coarse information structure, every matter sector contributes through its effect on the common equivalence network.

Gravity then couples not to one particular charge representation but to the total cost of persistent physical structure.

This could yield universality of free fall.

However, the model must demonstrate that different defect species distort the emergent metric according to the same energy measure.

If geometry depends on representation-specific details unrelated to total energy, the equivalence principle fails.

A finite universality test can compare two defect types with equal extracted energy but different automorphism charges.

Place each in otherwise identical configurations.

Measure the induced relational-distance distortion.

A gravitationally universal regime requires approximately equal geometric response.

This would be an important nontrivial result.

The thermodynamic route may also generate corrections to Einstein gravity.

Finite equivalence fibers, higher homotopy, nonlocal transport, or modified entropy-area relations could produce additional terms.

Possible deviations include:

higher-curvature corrections;

scale-dependent Newton coupling;

torsion;

nonlocal response;

entropy corrections;

or topology-dependent gravitational effects.

These should not be introduced speculatively before the leading Einstein-like limit is recovered.

The immediate goal is to establish whether the relational theory has the correct thermodynamic architecture.

The gravity conjecture can fail in several ways.

Equivalence entropy may not scale with area.

No universal effective temperature may emerge.

Energy flux may not couple consistently to geometric change.

The distance structure may remain species dependent.

Holonomy curvature may not correlate with metric curvature.

The continuum limit may produce a nongeometric phase.

The resulting field equation may differ fundamentally from Einstein's equation.

Any of these outcomes would require revising or abandoning the proposed gravity bridge.

The theory should treat them as genuine possibilities.

10.7 Status, Recovery Requirements, and Falsification

The constructions in this section range from exact finite definitions to speculative continuum hypotheses.

Their status must be separated clearly.

The relational distance:

d(v,w)
= inf_p C(p)

is an exact definition once the path cost is specified.

Whether it represents physical distance is a hypothesis tested by gauge invariance, universality, propagation, and scaling.

The effective dimension measures are standard calculations applied to the relational metric or diffusion process.

Whether the theory dynamically selects the observed dimensionality is an open question.

Holonomy curvature is exact in the finite gauge model.

Its identification with spacetime curvature requires frame, metric, and compatibility structures not yet fully derived.

Equivalence-fiber entropy can be defined exactly for finite groupoids.

Its identification with horizon entropy requires boundary, causal, and area-law results.

The thermodynamic gravity relation is a conjectural continuum bridge.

It is not established merely by writing:

delta Q = T delta S_eq.

A credible recovery of general relativity requires several explicit achievements.

First, an effective Lorentzian geometry must emerge.

The theory must produce:

a manifold-like continuum;

a metric with Lorentzian signature;

local causal cones;

and approximately invariant propagation speed.

Second, the relational connection must become compatible with this geometry.

A frame or coframe construction should connect identity transport with tangent structure.

Third, the effective curvature must be related to metric holonomy or an accepted generalized gravitational connection.

Fourth, matter and defect energy must define a conserved stress-energy tensor or equivalent source.

Fifth, the low-energy variational or thermodynamic equations must reduce to Einstein-like dynamics.

Sixth, the gravitational coupling must be finite and universal.

Seventh, the model must reproduce tested limits:

Newtonian gravity;

gravitational redshift;

light bending;

time dilation;

gravitational waves;

and cosmological expansion where applicable.

Eighth, corrections must remain within observational bounds or produce testable deviations.

The finite paper need not accomplish all of these.

It should specify which precursor result is actually obtained.

A minimal success for Section 10 would be:

a gauge-invariant relational metric;

agreement of two or more dimension estimators;

and a demonstrable correlation between defect energy and geometric distortion.

A stronger success would be:

frame-valued transport;

metric compatibility;

and a discrete curvature relation.

A still stronger success would be:

an entropy-area law and local thermodynamic response.

The full success would be:

a continuum Einstein-like field equation with controlled corrections.

The theory should also specify direct falsification criteria.

The emergent-distance proposal fails if d depends on gauge presentation.

It fails as a metric if zero-distance equivalence cannot be consistently quotiented or if the triangle inequality fails in a regime claimed to be ordinary space.

The universal-geometry proposal fails if different low-energy probes infer incompatible distances.

The dimension-selection proposal fails if the measured dimension is simply inherited from the chosen lattice.

The curvature bridge fails if holonomy cost and geodesic behavior remain unrelated.

The gravitational interpretation fails if equal-energy defects produce species-dependent geometric response.

The entropy proposal fails if gauge redundancy is counted as thermodynamic multiplicity.

The horizon bridge fails if equivalence entropy does not exhibit an area-dominated regime.

The thermodynamic derivation fails if no consistent temperature, energy flux, or focusing relation emerges.

The general-relativity recovery fails if the continuum equations do not approximate known gravitational phenomena.

These failure conditions distinguish a scientific program from an unrestricted analogy.

The computational program should proceed in stages.

Stage 1 assigns gauge-invariant edge costs on fixed finite complexes.

Stage 2 calculates relational shortest-path distances.

Stage 3 measures volume-growth and spectral dimensions.

Stage 4 introduces localized defects and measures metric distortion.

Stage 5 compares holonomy, graph, and geodesic curvature.

Stage 6 introduces frame and coframe data.

Stage 7 studies boundary entropy and automorphism-corrected subregion counting.

Stage 8 tests finite thermodynamic relations.

Stage 9 constructs a Lorentzian or causal extension.

Stage 10 investigates continuum gravitational equations.

GenesisBoard and related categorical simulations may contribute to the early stages by generating relational complexes and tracking:

path multiplicity;

coherence cost;

loop structure;

effective dimension;

and defect-induced changes.

However, benchmark calculations on controlled graphs and lattices remain necessary.

The central physical proposal of this section can be summarized as follows.

Local contexts do not occupy a preexisting space.

They become spatially related through the cost of coherent identification.

Low-cost transport defines proximity.

Families of low-cost paths define neighborhoods and effective dimension.

Path dependence defines curvature.

Stable obstruction defines matter.

Matter alters the cost and organization of transport.

The resulting backreaction changes effective geometry.

Coarse observational boundaries hide multiplicities of relational structure, producing entropy.

In a suitable continuum and thermodynamic limit, the coupled variation of geometry, energy, and equivalence entropy may reproduce gravitational dynamics.

This is the intended bridge from gauge coherence to spacetime.

At present, the bridge is a structured research program rather than a completed derivation.

Its value lies in making the required steps explicit.

The next section turns from formal construction to computation by specifying how GenesisBoard and exact finite models can implement the groupoid fields, gauge transformations, defects, partition functions, geometry measures, and falsification tests developed throughout the paper.

11. GenesisBoard Implementation and Computational Experiments

11.1 Extension of GenesisBoard

The mathematical constructions developed in the preceding sections require a computational environment capable of representing finite categories, groupoids, gauge transformations, higher coherence, defect sectors, and physical observables. GenesisBoard provides a natural starting

platform because it was designed to generate and measure compositional structure rather than merely simulate numerical fields on a conventional lattice.

The present implementation must nevertheless be extended substantially.

Earlier GenesisBoard models emphasized the growth of objects, morphisms, commuting triangles, alternative paths, diamond structures, path compression, and coherence witnesses. These quantities remain valuable, but Univalent Relational Gauge Dynamics requires additional explicit physical structure.

The extended platform should represent:

vertices carrying local identity objects;

oriented edges carrying admissible transport morphisms;

faces carrying ordered boundary data;

vertex automorphism groups;

local gauge transformations;

face and loop holonomies;

matter representations;

defect labels;

coherence actions;

physical observables;

and automorphism-weighted quantum sums.

The implementation should preserve a strict distinction between three levels.

The first level is the mathematical presentation.

This includes:

vertex labels;

edge identifiers;

groupoid-object names;

chosen matrix or permutation representations;

and data-storage conventions.

The second level is the physical equivalence structure.

This includes:

gauge transformations;

gauge orbits;

stabilizer groups;

conjugacy classes;

and groupoid isomorphisms.

The third level is the observable structure.

This includes:

action values;

Wilson loops;

defect classes;

mass-gap estimators;

correlation functions;

effective distances;

and dimensions.

The software must never infer physical distinction from raw labels alone.

Two configurations with different internal identifiers may belong to the same gauge orbit.

Two morphisms with different stored representations may implement the same abstract groupoid arrow.

Two lattices with different vertex numbering may represent the same relational complex.

Canonicalization and invariance tests are therefore central.

A minimal data model for the finite complex should include:

Vertex:
identifier,
identity-object type,
matter state,

boundary status,
optional geometric or causal metadata.

Edge:
identifier,
source vertex,
target vertex,
transport morphism,
orientation,
weight,
boundary status.

Face:
identifier,
ordered oriented boundary edges,
base vertex,
weight,
optional higher-coherence variable.

IdentityObject:
identifier,
groupoid component,
automorphism group,
representation spaces,
local cost,
admissibility metadata.

Morphism:
source object,
target object,
composition table or operator,
inverse,
representation map,
cost metadata.

Configuration:
object assignment,
edge assignment,
matter assignment,

boundary conditions,
sector labels.

The groupoid may be represented in several equivalent forms.

For small finite groupoids, the most transparent representation is an explicit table containing:

objects;

morphisms;

source and target;

identity arrows;

inverse arrows;

and composition.

For one-object groupoids, a finite group implementation is sufficient.

The group may be represented through:

a multiplication table;

permutations;

matrices;

group generators and relations;

or an external finite-group package.

For many-object connected groupoids, one practical construction begins with:

a representative group G;

a finite set of equivalent object presentations;

and chosen connecting isomorphisms.

For disconnected candidate sectors, use a disjoint union of such components.

The implementation should verify the groupoid axioms before simulation:

identity laws;

associativity;

inverse laws;

source-target compatibility;

and closure.

Invalid algebraic input must be rejected rather than silently propagated.

The complex representation should likewise be validated.

Every edge in a face boundary must join correctly to the next edge.

Every face boundary must close.

Orientation reversal must be implemented consistently.

Boundary and interior cells must be identified.

Global cycles should be computable through homology, fundamental-group approximations, or explicit cycle bases.

A configuration generator should support several modes.

Uniform sector mode assigns the same identity component to every connected region.

Equivalent-presentation mode assigns different objects from one connected groupoid component while preserving reversible transport.

Heterogeneous-sector mode permits multiple disconnected identity sectors separated by explicit interfaces.

Random transport mode assigns edge morphisms uniformly or according to a specified distribution.

Flat mode constructs assignments satisfying trivial face holonomy.

Defect-seeded mode inserts prescribed global holonomy, winding, or boundary conditions.

Matter-coupled mode assigns local representation values and potentials.

Higher-coherence mode assigns face 2-morphisms or crossed-module data.

The software should construct edge transports only when source and target objects admit the required morphism.

An attempted reversible transport between disconnected identity sectors must be:

rejected;

converted into an explicit interface process;

or assigned infinite action,

according to the selected model.

This prevents the simulation from treating genuine identity-changing processes as ordinary gauge transformations.

The gauge-transformation engine should assign:

g_v in Aut(A_v)

to every eligible vertex.

It must update edge transports by:

U_e -> g_t(e) o U_e o g_s(e)^-1,

and matter states by:

psi_v -> rho(g_v) psi_v.

The engine should support:

random gauge transformations;

complete enumeration for small systems;

gauge fixing;

orbit generation;

and stabilizer calculation.

Random gauge transformations serve as invariance tests.

Complete enumeration permits exact orbit classification.

Gauge fixing may accelerate computation but must not define the physical observables.

Orbit generation determines which raw configurations represent the same physical state.

Stabilizer calculation supplies the automorphism factor used in the partition function.

The holonomy engine should calculate:

face holonomy;

arbitrary closed-loop holonomy;

global-cycle holonomy;

represented traces;

conjugacy classes;

and spectra.

For non-Abelian groups, path ordering must be preserved exactly.

Changing the face basepoint should conjugate the result but leave class functions invariant.

Reversing orientation should invert the holonomy.

These relations should be included in automated unit tests.

The action engine should evaluate every term separately:

face coherence;

matter transport;

local potential;

identity-sector cost;

interface cost;

boundary cost;

defect cost;

and optional higher terms.

Reporting the components separately is important.

A total action value alone does not reveal what stabilizes a defect or selects a sector.

The output should make it possible to determine whether a result is driven by:

curvature;

matter mismatch;

identity maintenance;

topology;

or an inserted penalty.

The defect-analysis engine should identify sectors through user-selected invariants.

Possible classifiers include:

global holonomy class;

Abelian flux;

winding number;

domain-wall number;

homology class;

matter zero count;

cocycle class;

or connected component of configuration space under allowed local updates.

For small systems, all configurations and allowed update transitions can be represented as a meta-graph.

Its vertices are physical configurations.

Its edges are allowed local deformations.

Connected components of this meta-graph define dynamical sectors.

This construction gives an exact finite implementation of:

pi_0(Conf_phys).

For larger systems, sector labels must be inferred through topological invariants and conserved quantities.

The extended GenesisBoard should also represent temporal layers.

A space-time complex may be constructed from repeated spatial slices.

Temporal edges transport identity and matter from one slice to the next.

Temporal plaquettes contribute to the action.

This supports:

transfer matrices;

correlation functions;

defect worldlines;

and mass-gap calculations.

The first implementation should remain deliberately modest.

It should use:

small finite groups;

small regular complexes;

exact arithmetic where possible;

and full enumeration.

Only after every invariant and counting identity has been verified should the software move to large stochastic simulations.

The first purpose of GenesisBoard is not speed.

It is correctness, transparency, and the discovery of which structures are worth scaling.

11.2 Baseline Experiments

Every new model should be compared with controlled baselines.

Without baselines, an apparent defect, phase transition, preferred group, or effective dimension may arise from coding conventions, finite-size effects, or generic random-network behavior rather than the proposed coherence dynamics.

The baseline program should proceed from the simplest possible systems toward the full heterogeneous model.

The first baseline is the trivial group model.

Let the identity object have only the identity automorphism.

Every edge transport is then trivial.

All face holonomies equal the identity.

The face action vanishes.

Any remaining structure must arise from:

matter variables;

identity-sector costs;

boundary conditions;

or the combinatorics of the complex.

This baseline tests whether purported gauge effects persist when gauge structure is absent.

The second baseline is the one-object Abelian model.

The smallest useful example is Z_2.

Every oriented edge carries:

$U_e = +1$ or -1.

Face holonomy is the product around the boundary.

The exact configuration space, gauge orbits, stabilizers, and partition function can be calculated on small complexes.

This model tests:

orientation handling;

gauge transformations;

flux sectors;

Wilson loops;

and orbit counting.

The third baseline is a finite cyclic group Z_n.

This introduces multiple discrete phase sectors while preserving commutativity.

It tests:

phase-like holonomy;

charge characters;

and winding behavior.

The fourth baseline is a finite non-Abelian group such as S_3.

This introduces:

noncommuting edge transports;

nontrivial conjugacy classes;

path ordering;

and higher-dimensional representations.

The calculation should confirm that:

face holonomies change by conjugation;

class functions remain invariant;

and loop observables depend on ordered composition.

The fifth baseline is a known lattice gauge model.

For each chosen group, compare GenesisBoard results with exact or published finite-lattice calculations where available.

The comparison should include:

partition functions;

mean plaquette action;

Wilson-loop expectation values;

specific heat;

and sector degeneracies.

Agreement establishes that the one-object recovery theorem is implemented correctly.

The sixth baseline is a flat random connection.

Generate random vertex frames g_v and construct edge variables:

U_e = g_t(e) g_s(e)^-1.

Every contractible face then has trivial holonomy.

The raw edge assignment may look highly irregular, but it is pure gauge.

The action and all physical curvature observables must equal those of the trivial configuration.

This is one of the most important tests of presentation independence.

A model that assigns high coherence cost to such a pure-gauge configuration is measuring descriptive complexity rather than physical curvature.

The seventh baseline is random transport without coherence optimization.

Assign edge morphisms independently and uniformly.

Measure:

holonomy distribution;

action;

defect density;

effective distance;

and path multiplicity.

This establishes the disordered reference ensemble.

Coherence-driven simulations should be compared against this null model.

The eighth baseline is the coherence-only model.

Use the face action with no matter, no identity-sector variation, and no explicit defect term.

This reveals whether the action alone creates:

flat phases;

flux phases;

confining behavior;

or localized curvature.

The ninth baseline adds matter but retains one fixed group.

This tests whether familiar vortex, domain, or symmetry-breaking structures arise before heterogeneous identity sectors are introduced.

Any claimed new defect should be compared with known defects of the recovered gauge-matter theory.

The tenth baseline is a presentation-multiplicity test.

Represent one abstract groupoid component using:

one object;

several equivalent objects;

and several relabelings of those objects.

The physical partition function and observables should agree after groupoid weighting.

If adding redundant object presentations changes the result, the implementation is overcounting description.

The eleventh baseline is a sector-prior test.

For heterogeneous candidate groups, compare:

equal prior weight;

group-order penalties;

description-length penalties;

and alternative normalizations.

This shows whether preferred-sector results are robust or prior driven.

The twelfth baseline is a lattice-structure test.

Run the same physical model on:

triangular;

square;

irregular;

and randomly relabeled complexes

with comparable size and topology.

Quantities claimed to be universal should converge after appropriate normalization.

The thirteenth baseline is a topology test.

Compare:

disk;

sphere;

cylinder;

torus;

and punctured surfaces.

This separates:

local curvature;

boundary effects;

and global holonomy.

A nontrivial loop sector on a torus should disappear or change appropriately on a simply connected disk.

The fourteenth baseline is a boundary-condition test.

Compare:

fixed;

free;

periodic;

flux-imposing;

and mixed boundaries.

Defects forced by boundaries should not be misidentified as spontaneously generated particles.

The fifteenth baseline is a random-label test.

Randomly permute:

vertex identifiers;

edge identifiers;

group-element labels;

object names;

and representation bases.

All physical outputs must remain unchanged.

The sixteenth baseline is an update-rule test.

Use several valid Monte Carlo or relaxation algorithms:

Metropolis;

heat bath;

cluster updates where available;

simulated annealing;

and exact enumeration on small systems.

Equilibrium observables should agree within statistical error.

Dependence on the update algorithm may reveal poor equilibration or hidden nonergodicity.

The seventeenth baseline is an ablation series.

Remove one structural ingredient at a time:

face coherence;

matter transport;

identity cost;

interface cost;

automorphism weighting;

topological constraint;

or higher cells.

Measure which claimed phenomena disappear.

This is necessary for causal interpretation.

If a preferred gauge sector remains when the coherence term is removed, coherence cannot be credited for the selection.

If a defect persists after topology and matter winding are removed, another mechanism is responsible.

Each baseline should be associated with a predefined expectation.

For example:

pure-gauge flat assignments must have zero face action;

gauge-transformed configurations must have identical observables;

trivial-group models must show no gauge holonomy;

one-object models must reproduce lattice gauge theory;

redundant presentations must not alter physical partition functions;

and nontrivial global sectors must depend on topology.

These expectations should be written before the simulations are run.

The baseline phase of the project may appear less exciting than searching immediately for a ToE-like result, but it is indispensable.

A theory that cannot recover and distinguish its simplest controls cannot support credible claims about emergent matter, geometry, or gauge selection.

11.3 Primary Observables

The computational implementation should produce a standardized set of observables for every run. This common output permits direct comparison across groups, complexes, couplings, defect sectors, and model extensions.

The first observable is the total coherence action:

S_total = S_URD(Phi).

The simulation should also report each component:

S_face;

S_matter;

S_potential;

S_identity;

S_interface;

S_boundary;

S_defect;

and S_higher where applicable.

Component separation identifies the physical origin of a configuration's cost.

The second observable is mean face-coherence cost:

E_face
= 1 / |F|
sum over f of K(H_f).

The distribution of face costs should also be recorded.

A mean value may conceal localized defects or coexistence between flat and highly curved regions.

Useful summaries include:

minimum;

maximum;

variance;

histogram;

and spatial correlation.

The third observable is the holonomy spectrum.

For each face and selected global loop, record:

the group element or morphism class;

the conjugacy class;

representation traces;

eigenvalue spectrum;

and orientation-reversed value.

For finite groups, report the frequency of each conjugacy class.

For Abelian groups, report the flux or phase distribution.

The fourth observable is Wilson-loop behavior.

For closed loops gamma of different sizes and shapes, compute:

W_gamma
= Tr rho(H_gamma).

Average over equivalent placements where appropriate.

Study whether:

-log <W_gamma>

scales primarily with enclosed area, perimeter, topology, or another measure.

This distinguishes confinement-like, screened, and topological regimes.

The fifth observable is gauge-orbit structure.

Record:

number of raw configurations sampled or enumerated;

number of physical gauge orbits;

orbit-size distribution;

stabilizer-size distribution;

and number of enhanced-symmetry configurations.

This information is central to the groupoid interpretation.

The sixth observable is the automorphism-weighted state count.

For a configuration groupoid X:

|X|_h
= sum over [Phi]
1 / |Aut(Phi)|.

For action-weighted systems, calculate:

Z
= sum over [Phi]
exp[-S(Phi)/hbar]
/ |Aut(Phi)|.

Compare these with:

ordinary orbit count;

raw presentation count;

and unweighted quotient count.

The differences quantify the effect of symmetry correction.

The seventh observable is defect classification.

For every configuration, record:

defect label;

topological charge;

global holonomy;

winding number;

domain-wall number;

or higher obstruction class.

The fraction of configurations in each sector gives:

P(d) = Z_d / Z.

The eighth observable is defect localization.

Define local excess action density relative to the vacuum.

Measure:

defect center;

radius;

inverse participation ratio;

fraction of excess action within a chosen radius;

and dependence on system size.

The inverse participation ratio may be defined schematically as:

IPR
= sum_x epsilon_x^2 /
(sum_x epsilon_x)^2.

A localized defect has a larger IPR than a uniformly distributed excitation, after volume normalization.

The ninth observable is defect lifetime.

Under real-time, stochastic, or relaxation dynamics, measure how long a defect label and localized profile persist.

For metastable sectors, estimate:

decay rate;

barrier height;

and dependence on temperature or coupling.

The tenth observable is the mass-gap estimator.

Construct gauge-invariant operators O_r in selected charge and defect sectors.

Calculate Euclidean correlation functions:

C_r(t)
= <O_r(t) O_r(0)^dagger>.

Extract effective masses from ratios or fits.

Report:

finite-size gap;

operator dependence;

fit interval;

uncertainty;

and scaling behavior.

The eleventh observable is the charge representation.

For each stable defect, determine:

automorphism-group action;

character values;

irreducible-representation decomposition;

and transformation under fusion.

Verify that the representation class remains unchanged under gauge transformations and defect motion.

The twelfth observable is sector free energy.

For candidate identity sector i:

$$F_i = -k_B T \log Z_i.$$

Report differences:

$$\Delta F_{ij} = F_i - F_j.$$

The sign and scaling of these differences determine sector dominance.

The thirteenth observable is sector probability:

$$P_i = Z_i / \sum_j Z_j,$$

including stated prior weights.

Report sensitivity to:

prior choice;

normalization;

lattice size;

and matter content.

The fourteenth observable is correlation length.

For gauge-invariant two-point functions:

C(r)
approximately proportional to
exp(-r/xi),

extract xi.

Compare xi with the inverse mass gap.

The fifteenth observable is effective relational distance.

For each configuration or ensemble average, calculate:

d(v,w)
= inf_p C(p).

Record:

distance matrix;

geodesic multiplicity;

diameter;

mean distance;

and zero-cost equivalence classes.

The sixteenth observable is volume-growth dimension.

For selected vertices:

N_v(R)
= number or weighted measure of points within relational radius R.

Estimate:

d_vol(R)
= d log N_v / d log R.

The seventeenth observable is spectral dimension.

Run diffusion on the relational network.

Calculate return probability:

P_return(t),

and estimate:

d_spec(t)
= -2 d log P_return / d log t.

The eighteenth observable is curvature comparison.

For each region, compare:

holonomy cost;

graph curvature;

Regge-like deficit where defined;

and geodesic deviation.

Calculate correlations among these measures.

The nineteenth observable is boundary entropy.

For selected regions, compute restricted partition functions compatible with fixed boundary data.

Estimate:

S_boundary;

area scaling;

volume scaling;

topological corrections;

and automorphism contributions.

The twentieth observable is gauge-invariance error.

For random gauge transformations, compute:

Delta S
= S(g · Phi) - S(Phi),

and analogous differences for every claimed physical observable.

The expected value is exactly zero in exact arithmetic or within machine tolerance numerically.

This should be reported routinely rather than only during debugging.

The twenty-first observable is composition error.

For path transports, verify:

$U_{(q \circ p)} = U_q \circ U_p$.

For amplitudes, test the gluing relation through intermediate boundaries.

For higher structures, test associators and coherence laws.

The twenty-second observable is refinement stability.

Compare physical quantities across subdivisions of the same macroscopic complex.

Record:

renormalized action density;

defect energy;

mass ratios;

distance;

dimension;

and sector probabilities.

The twenty-third observable is computational complexity.

Report:

runtime;

memory use;

number of configurations;

number of orbits;

sampling autocorrelation;

and effective sample size.

This is necessary for evaluating which parts of the program can scale.

The observables should be stored in a machine-readable standard format.

Every run should include:

model version;

code commit identifier;

date and time;

random seed;

host information;

complex specification;

groupoid specification;

couplings;

boundary conditions;

update algorithm;

number of sweeps;

and convergence diagnostics.

This metadata is part of scientific reproducibility.

A summary score should not replace the full observable set.

Earlier GenesisBoard work often used aggregate coherence measures to compare runs. Such measures remain useful for exploration, but the physical theory requires decomposed observables with explicit interpretation.

A high coherence score alone does not establish:

gauge invariance;

defect stability;

mass;

dimension;

or gravity.

Every claimed phenomenon must be tied to its appropriate invariant measurement.

11.4 Gauge-Invariance Tests

Gauge invariance is the first nonnegotiable computational requirement.

A configuration and any of its admissible gauge transforms must produce exactly the same physical outputs.

The test protocol begins by generating or loading a configuration Phi.

Generate a random local gauge transformation:

g = {g_v}.

Construct:

Phi' = g · Phi.

Then compare:

S_URD(Phi');

face-action components;

Wilson loops;

defect labels;

sector labels;

correlation operators;

distance measures;

and partition-function contribution.

For every gauge-invariant scalar observable O:

O(Phi') - O(Phi) = 0.

For covariant quantities, verify the expected transformation law instead of equality.

For example:

D_e psi'
= rho(g_t(e)) D_e psi.

For face holonomy:

H_f'
= g_vf H_f g_vf^-1.

For open path transport:

U_p'
= g_t(p) U_p g_s(p)^-1.

The test suite should include random and exhaustive modes.

For small groups and complexes, enumerate every local gauge transformation.

For larger systems, sample many random transformations.

Exhaustive tests are particularly valuable because one missed source-target convention can produce failures only for certain combinations.

The gauge-invariance tests should be performed at several levels.

Level 1 tests algebraic identities for one edge and one path.

Level 2 tests face holonomy.

Level 3 tests the complete action.

Level 4 tests defect classification.

Level 5 tests orbit and stabilizer enumeration.

Level 6 tests partition-function equivalence.

Level 7 tests derived geometry.

Level 8 tests correlation functions and transfer matrices.

The one-edge test verifies:

U_e^-1 transforms consistently;

source and target automorphisms occur on the correct sides;

and matter fields transform in the correct representation spaces.

The face test verifies intermediate cancellation around closed loops.

The action test verifies invariance term by term.

The defect test verifies that gauge-related representatives retain the same obstruction label.

The orbit test verifies that gauge transformations partition the raw configuration set correctly.

The partition test verifies:

1 / |Gauge|
sum over raw Phi e^(-S(Phi)/hbar)

equals:

sum over [Phi]
e^(-S(Phi)/hbar)
/ |Aut(Phi)|.

The geometry test requires special care.

If the relational distance is derived from raw edge variables, it may accidentally become gauge dependent.

The test should compare full distance matrices before and after gauge transformation.

Any change reveals that the proposed cost is not physical.

The correlation test verifies that gauge-invariant operators yield identical correlators after transforming every configuration and boundary state consistently.

Gauge fixing should be tested independently.

Choose several gauge-fixing procedures:

spanning-tree gauge;

maximal-tree gauge;

Landau-like optimization where meaningful;

or canonical orbit representative selection.

Calculate physical observables in each gauge.

They must agree.

Gauge-fixed raw quantities may differ, but all final results must coincide.

Gribov-like multiplicity may occur in optimization-based gauge fixing.

The physical calculation should not depend on which local optimum is reached.

Exact orbit methods on small systems provide the benchmark.

Boundary transformations must be separated into two classes.

Interior gauge transformations vanish or are fixed on the boundary.

Boundary symmetries act nontrivially on boundary states.

An observable may be invariant under the first but transform under the second.

The software should label these transformation classes explicitly.

Otherwise, a genuine boundary charge may be incorrectly classified as a gauge failure.

The test tolerance should be specified.

For finite groups and exact rational or symbolic representations, invariance should be exact.

For floating-point matrix groups, report:

absolute error;

relative error;

and maximum deviation across tests.

The error should scale with numerical precision rather than system size in an uncontrolled manner.

Gauge-invariance testing should be continuous during development.

Every new action term, observable, or update move should be rejected until it passes the test suite.

This is especially important for:

identity-sector interface terms;

matter potentials;

relational-distance functions;

and entropy calculations.

These are the places where presentation dependence can enter unnoticed.

The completed paper should include a table of invariance tests.

For each observable, report:

transformation type;

expected behavior;

maximum measured error;

number of tests;

and pass criterion.

This provides stronger evidence than a general statement that the code is gauge invariant.

11.5 Scaling Tests and Continuum Tendencies

Finite models can demonstrate exact structure, but physical relevance depends on behavior across increasing size and refinement.

A small complex may display stable defects, nonzero gaps, preferred sectors, or apparent dimensions that disappear in larger systems.

Scaling tests determine which phenomena survive.

Several distinct limits must be separated.

The thermodynamic limit increases the number of cells while keeping lattice spacing fixed:

L -> infinity.

The continuum limit decreases the lattice spacing:

a -> 0,

while physical distances and renormalized quantities are held fixed.

The large-time limit increases Euclidean temporal extent to isolate ground-state behavior.

The large-complexity limit enlarges the candidate groupoid or category.

The coarse-graining limit integrates out microscopic relational data.

These limits need not commute.

A result should state which limit is being studied.

The first scaling test concerns action density.

For an extensive homogeneous phase:

S_total / |F|

or:

S_total / Volume

should approach a stable value.

Strong growth or decay may indicate incorrect normalization.

The second test concerns Wilson loops.

Measure loops of increasing size.

Determine whether:

-log <W_gamma>

scales with area;

perimeter;

or topology.

The scaling behavior characterizes the gauge phase.

The third test concerns defect energy.

For a localized defect:

Delta E_d(L)

should approach a finite value as the system size becomes much larger than the defect radius, apart from controlled finite-volume corrections.

If Delta E_d grows with volume, the defect may carry an unscreened global field or be delocalized.

If it vanishes, the defect may not survive.

The fourth test concerns defect radius.

Measure:

R_d(L,a).

A physical localized defect should satisfy:

R_d / L -> 0

in the thermodynamic limit.

In a continuum limit, its physical radius:

a R_d

should approach a finite value or follow a known scaling law.

The fifth test concerns mass gaps.

Extract:

Delta(L,a).

Fit finite-size corrections.

Possible forms include:

Delta(L)
= Delta_infinity + c exp(-L/xi);

Delta(L)
= Delta_infinity + c/L^p;

or critical scaling:

Delta(L)
approximately proportional to
L^(-z).

A positive Delta_infinity supports a massive phase.

The sixth test concerns mass ratios.

Dimensionless ratios:

Delta_i / Delta_j

are particularly important because they may converge even when the overall scale changes.

A stable ratio generated without independent mass parameters would be a strong result.

The seventh test concerns sector free-energy density.

For sector i:

f_i(L)
= F_i(L) / Volume.

Selection requires stable differences:

Delta f_ij
= f_i - f_j.

A total free-energy difference that remains finite while volume grows may represent only a boundary preference rather than bulk selection.

The eighth test concerns effective dimension.

Measure:

d_vol(R,L);

d_spec(t,L);

and d_embed.

Identify a scale window where finite-size and lattice effects are small.

The width of this window should increase with system size if a continuum geometric phase exists.

The ninth test concerns isotropy.

Compare correlation lengths and propagation in different lattice directions.

Anisotropy should decrease after renormalization if rotational symmetry emerges.

The tenth test concerns gauge-invariance error with size.

Numerical error should not grow so rapidly that physical invariance becomes unreliable.

The eleventh test concerns autocorrelation and sampling.

Near phase transitions, Monte Carlo chains may become trapped.

Report:

integrated autocorrelation time;

independent sample count;

replica consistency;

and hysteresis.

The twelfth test concerns topology.

Repeat scaling on several topologies.

Local continuum quantities should become insensitive to global topology far from boundaries, while global holonomies and topological sectors should retain appropriate dependence.

The thirteenth test concerns subdivision invariance.

Take a complex and refine each face into smaller faces.

Map a coarse configuration into the refined model.

Compare:

coarse and fine holonomy;

action;

distance;

and defect label.

The refined action should reproduce the coarse physical behavior after coupling rescaling.

The fourteenth test concerns renormalization.

Block neighboring cells into coarse cells.

Construct effective transport and identity variables.

Measure how the couplings transform.

The renormalization map may take:

beta_g -> beta_g';

kappa -> kappa';

identity costs -> effective identity costs;

and higher operators -> generated terms.

A continuum limit corresponds to a scaling trajectory approaching a fixed point or controlled critical surface.

The equivalence-first theory should examine whether coarse-graining preserves the groupoid structure.

Microscopic identity objects may combine into effective objects.

Their automorphism groups may change.

New gauge redundancy may emerge.

Higher morphisms may become relevant.

This is one possible route to emergent gauge structure.

The fifteenth test concerns universality.

Run different microscopic actions that share:

symmetry;

dimensionality;

and long-range structure.

If they approach the same critical behavior and observable ratios, the continuum theory is universal.

A ToE claim cannot rest entirely on one finely tuned lattice action unless a reason for that action is derived.

The sixteenth test concerns continuum recovery of Yang-Mills behavior.

In the one-object sector, compare scaling with established lattice gauge theory.

The broader model should reproduce known beta functions, correlation behavior, or phase properties in the appropriate limit.

The seventeenth test concerns geometry backreaction.

Increase defect energy while holding charge type fixed.

Measure how relational distances and curvature indicators scale.

A gravitational interpretation requires a stable relation rather than arbitrary distortion.

The eighteenth test concerns entropy-area behavior.

For subregions of increasing size, fit:

S(R)
= alpha Area(R)

- beta Volume(R)
- gamma log Area(R)
- constant
- corrections.

Determine which term dominates in the relevant phase.

An area law should not be claimed from a few small regions.

The nineteenth test concerns causal propagation.

In a temporal model, measure the maximum or dominant propagation speed of disturbances.

Test whether a stable effective cone emerges and whether different defect species share it.

The twentieth test concerns higher-cell refinement.

Add 3-cells or 2-morphism data.

Determine whether the 1-groupoid observables remain stable or require modification.

The scaling program should use predefined model families.

For example:

square lattices of size L x L;

periodic strips of spatial size L and temporal extent T;

triangulated spheres with increasing refinement;

random complexes with controlled degree distribution;

and GenesisBoard-generated complexes selected by coherence statistics.

Every reported continuum tendency should include:

the sequence of complexes;

coupling scaling;

fit model;

uncertainty;

alternative fits;

and excluded small-size points.

Visual resemblance to convergence is insufficient.

Statistical and systematic errors must be reported.

A negative scaling result is scientifically valuable.

A defect that disappears under refinement should be identified as a finite artifact.

A preferred sector whose free-energy difference vanishes should not be promoted as fundamental.

A dimension that follows the imposed lattice should not be called emergent.

The scaling tests are where the theory moves from finite demonstration to possible physical law.

11.6 Exact Enumeration, Sampling, and Optimization

Different regions of the model require different computational methods.

Exact enumeration is the gold standard for small systems.

It should be used whenever the raw configuration count is manageable.

For a one-object finite group model with |E_plus| independent edges:

N_raw = |G|^|E_plus|.

Matter and multiple identity objects multiply this count.

Even modest complexes become large quickly.

Nevertheless, exact enumeration on very small systems is essential for validating every later approximation.

The exact workflow is:

generate every admissible raw configuration;

calculate its action and observables;

generate its gauge orbit;

select or compute its physical class;

calculate its stabilizer;

accumulate the raw and groupoid partition sums;

and verify all counting identities.

A canonical representative may reduce repeated work.

Possible canonicalization methods include:

lexicographic minimization over the gauge orbit;

spanning-tree gauge;

graph-isomorphism canonical labels;

or hash values of invariant data supplemented by orbit checks.

A canonicalization method must not merge physically distinct configurations that share the same simple invariants.

Wilson loops and action values alone do not uniquely identify a gauge orbit in general.

For larger finite systems, Monte Carlo sampling is required.

A basic Metropolis update proposes a change to one edge transport:

U_e -> U_e'.

Only the faces and matter terms incident on e need to be recalculated.

The proposal is accepted with probability:

min[1, exp(-Delta S/hbar)]

in the Euclidean quantum form.

For finite-temperature simulations, replace hbar by the appropriate thermal factor.

Matter variables and identity objects require separate update moves.

An object-sector update may require simultaneous changes to all incident edge morphisms.

Naive single-vertex proposals may have low acceptance because most neighboring transports become inadmissible.

Cluster or coordinated updates may be necessary.

Heat-bath algorithms can sample a local variable from its exact conditional distribution where available.

For Abelian and small finite groups, this may be efficient.

Cluster algorithms may reduce critical slowing down in certain matter sectors.

Worm algorithms may sample flux and defect configurations directly.

Parallel tempering may help transitions among separated identity sectors.

Multicanonical sampling may estimate free-energy barriers and rare defects.

The update set must be ergodic within the intended physical sector.

If the simulation cannot move between all configurations of a sector, equilibrium averages may depend on initial conditions.

Conversely, updates should not cross a conserved topological sector when a restricted partition function is being calculated.

The software should provide both:

sector-preserving updates;

and explicit sector-changing updates.

This distinction permits calculation of metastability and transition rates.

Gauge transformations can be inserted as zero-action updates.

They should not affect physical observables.

Including them may improve exploration of raw presentation space but does not move the system between physical states.

A simulation that spends most of its time performing gauge changes gains little physical sampling.

Gauge fixing or orbit-aware updates may be more efficient.

The choice depends on the algorithm.

Optimization methods are used to find minimum-action configurations and defect profiles.

Possible methods include:

steepest descent for continuous variables;

coordinate descent;

simulated annealing;

genetic search;

integer optimization;

dynamic programming on simple complexes;

and exhaustive sector minimization.

For finite groups, local descent may become trapped in metastable minima.

Multiple random starts and annealing schedules are required.

A claimed ground state should be verified by exact enumeration where possible or bounded through independent methods.

Defect minimum calculations should constrain the sector label explicitly.

Otherwise the optimizer may simply remove the defect and return to the vacuum.

Sector constraints can be enforced through:

fixed boundary winding;

fixed global holonomy;

topological update restrictions;

or penalty terms used only as computational constraints.

Any penalty contribution must be removed when reporting the physical defect energy unless it represents an actual part of the model.

Transfer matrices are especially useful for narrow spatial lattices.

Enumerate all physical or gauge-fixed states on one spatial slice.

Calculate transition weights through one temporal slab.

Construct the matrix T.

Diagonalize T to obtain:

ground-state energy;

mass gaps;

charge-sector spectra;

and correlation lengths.

The transfer matrix offers exact finite quantum information and should be used before relying on noisy long-time correlation fits.

Tensor-network methods may extend the calculation to larger systems.

Local group and groupoid constraints can be encoded in tensors.

Contraction yields partition functions and observables.

Symmetry-aware tensor networks can preserve gauge invariance explicitly.

The groupoid generalization may require tensors whose indices carry object and morphism labels.

This is a promising direction for scaling exact-like calculations.

GenesisBoard’s existing multi-worker architecture may support stochastic searches and parameter sweeps.

Different workers can explore:

random seeds;

groups;

couplings;

boundary conditions;

and defect sectors.

However, worker outputs should be treated as independent samples or optimization attempts, not as evidence of reproducibility merely because many runs were performed.

The same phenomenon should be reproduced across:

different seeds;

different hosts;

different algorithms;

and controlled model variants.

The computational engine should separate exploratory and confirmatory modes.

Exploratory mode searches broad parameter ranges and identifies possible transitions, defects, or preferred sectors.

Confirmatory mode uses predefined parameters, larger statistics, independent seeds, and frozen analysis scripts.

Results discovered and evaluated on the same runs are vulnerable to selection bias.

A separate confirmation stage is required.

The software should record convergence diagnostics.

For Monte Carlo runs:

burn-in length;

autocorrelation time;

acceptance rate;

replica agreement;

and effective sample size.

For optimization:

number of starts;

best and median minimum;

convergence criterion;

and frequency of reaching the best solution.

For transfer matrices:

matrix dimension;

symmetry blocks;

numerical precision;

and residual eigenvalue error.

For exact enumeration:

configuration count;

orbit count;

and checksum identities.

The algorithms should be benchmarked against one another.

On small systems:

exact enumeration;

Monte Carlo;

transfer matrix;

and tensor contraction

should agree.

This cross-validation is more convincing than any single implementation.

11.7 GenesisBoard as a Relational Gauge Laboratory

GenesisBoard differs from a conventional lattice simulator in its ability to study the growth and organization of compositional structure.

This capability may reveal phenomena that are hidden when the complex is fixed.

The lattice need not remain static.

Vertices, edges, faces, and higher cells may be generated dynamically according to local relational rules.

A dynamic GenesisBoard model may allow:

creation of new identity contexts;

formation of new transport morphisms;

addition of coherence faces;

compression of equivalent paths;

generation of higher witnesses;

and removal of redundant structure.

The evolving complex itself becomes part of the physical configuration.

This permits study of:

emergent locality;

effective dimension;

phase transitions in compositional structure;

and geometry generated by coherence.

However, dynamic-complex simulations introduce additional ambiguity.

A newly created edge may change both the field and the geometry.

A new face may impose a coherence constraint that did not previously exist.

Growth rules can strongly bias the outcome.

The model must distinguish:

physical dynamics;

algorithmic construction;

and measurement of the generated structure.

A dynamic rule should be stated as explicitly as an action.

One option is a local stochastic growth process satisfying detailed balance with respect to an extended action:

S_extended
= S_URD

- S_complexity
- S_topology
- S_growth.

Moves may include:

add or delete an edge;

add or delete a face;

split or merge a vertex;

refine or coarse-grain a region;

and add or remove higher coherence witnesses.

Each move changes the state count and must include the appropriate proposal and measure factors.

Another option is a nonequilibrium generative model.

Such a model may be useful for studying cosmological growth but does not automatically define an equilibrium quantum theory.

The distinction should be explicit.

GenesisBoard's path multiplicity measures may be reinterpreted physically.

For two contexts v and w, count the distinct admissible transport paths.

Classify them by:

holonomy;

action;

homotopy;

and higher equivalence.

The weighted path count may influence:

transition amplitudes;

entropy;

and effective proximity.

A large number of low-cost coherent paths may make two contexts effectively near even when their shortest combinatorial path is long.

This suggests a path-ensemble distance:

d_path(v,w)
= -ell_0 log
sum over p:v->w exp[-C(p)/ell_0],

or a quantum analogue with complex phases.

At low effective temperature, the shortest path dominates.

At higher path entropy, many routes contribute.

This combines distance and multiplicity in a natural way.

GenesisBoard's compression statistics may also acquire physical meaning.

If many distinct paths are identified by coherence relations, the effective relational description becomes simpler.

A compression transition may indicate emergence of:

a stable quotient;

a gauge sector;

or a low-dimensional geometry.

However, compression must be based on admissible physical equivalence, not arbitrary graph simplification.

The higher-cell witnesses generated by GenesisBoard can represent:

commuting diagrams;

2-morphisms;

associators;

and coherence relations.

Their density and topology may distinguish:

ordinary gauge phases;

higher gauge phases;

and anomalous or frustrated phases.

A face may witness equality or higher equivalence between two path composites.

Collections of faces may form closed surfaces.

Three-dimensional coherence cells may relate alternative surface compositions.

This is the natural computational route from groupoid gauge theory to higher gauge theory.

A GenesisBoard experiment can compare four regimes.

Regime 1 contains only objects and edges.

Path dependence is unconstrained.

Regime 2 adds faces that penalize or witness loop coherence.

Ordinary gauge curvature emerges.

Regime 3 adds 2-morphisms among path transports.

Higher gauge equivalence emerges.

Regime 4 adds dynamic higher cells and local-to-global gluing.

A higher relational geometry may emerge.

The program should identify what new observables become possible at each regime.

The dynamic model can also investigate whether stable defect structures arise spontaneously during category growth.

Track:

when a nontrivial loop first appears;

whether it gains a face;

whether its holonomy becomes stable;

whether surrounding paths reorganize;

and whether the structure persists under continued growth.

A persistent local obstruction may behave as an emergent particle within the evolving network.

Its identity can be tracked through:

defect label;

automorphism representation;

and transport between successive network states.

This is a computational realization of objecthood as persistence across transformations.

The strongest GenesisBoard result would be a reproducible phase transition in which:

a random compositional network develops coherent local groupoid structure;

a preferred automorphism sector emerges;

stable defect excitations appear;

and an effective geometry with stable dimension forms.

Such a result would not by itself establish fundamental physics.

It would demonstrate that the proposed structures can coemerge from simple relational rules.

The simulation should be compared against random graph, random category, and fixed-lattice controls.

The role of every growth rule should be tested through ablation.

11.8 Quantum and Statistical Experiments

The finite model permits exact quantum and statistical experiments unavailable in a purely conceptual framework.

The first experiment is complete orbit quantization on a single face.

Choose a finite group G.

Enumerate all edge assignments.

Calculate:

face holonomy;

gauge orbit;

stabilizer;

action;

and exact contribution to Z.

Verify the equivalence between raw and groupoid sums.

The second experiment uses two faces sharing an edge.

This is the smallest system in which local curvature terms interact.

Measure correlations between neighboring plaquettes.

The third experiment uses a periodic strip with a temporal direction.

Construct the transfer matrix.

Extract:

ground state;

first excitation;

charge sectors;

and finite mass gap.

The fourth experiment introduces a noncontractible spatial cycle.

Measure global holonomy sectors and tunneling between them.

The fifth experiment introduces matter winding and a localized defect.

Calculate the defect profile and two-point correlation function.

The sixth experiment compares two gauge sectors in a heterogeneous groupoid.

Calculate exact sector probabilities and determine whether any preference survives normalization.

The seventh experiment introduces redundant equivalent object presentations.

Verify that physical Z remains unchanged.

The eighth experiment implements a two-route interference geometry.

Calculate Lorentzian amplitudes for two identity-transport histories.

Insert a holonomy between the paths and measure the phase shift.

The ninth experiment couples the routes to an environment.

Measure suppression of off-diagonal interference and the formation of a stable outcome record.

The tenth experiment calculates subregion entropy.

Compare:

raw state count;

gauge-quotiented count;

automorphism-weighted count;

and quantum entanglement where a Hilbert-space formulation is available.

The eleventh experiment tests defect fusion.

Prepare two defects.

Bring them together through controlled updates.

Measure:

combined charge;

energy;

binding;

and possible annihilation.

The twelfth experiment tests equal-energy geometric response.

Construct two defect species with different charge representations but matched energy.

Measure induced relational-distance distortion.

This is an early test of gravitational universality.

The thirteenth experiment tests dimensional flow.

Generate or weight complexes at several coherence couplings.

Measure spectral dimension as a function of diffusion scale.

The fourteenth experiment tests sector selection under matter coupling.

Compare the preferred pure gauge sector with the preferred gauge-plus-matter sector.

This determines whether matter stabilizes a different automorphism structure.

The fifteenth experiment tests higher coherence.

Add face 2-morphisms or crossed-module variables.

Compare the resulting defect classes and partition functions with the 1-groupoid model.

Every experiment should have:

a primary hypothesis;

a null hypothesis;

an observable;

an acceptance criterion;

and a failure interpretation.

For example:

Primary hypothesis:
A topological defect sector has a positive scaling mass gap.

Null hypothesis:
The apparent gap is a finite-size artifact and tends to zero.

Observable:
Gauge-invariant two-point correlation decay across increasing L.

Acceptance criterion:
Extrapolated gap remains positive with uncertainty excluding zero under multiple fit forms.

Failure interpretation:
The defect does not define a massive continuum excitation.

This structure prevents retrospective reinterpretation.

11.9 Formal Verification and Proof-Assisted Computation

Some parts of the model are suitable for formal verification.

A proof assistant can encode:

finite categories;

groupoids;

group actions;

gauge transformations;

path composition;

holonomy;

and invariance theorems.

The first formal target is the conjugation law for closed-loop holonomy.

The second is gauge invariance of the face action.

The third is covariance of the matter difference.

The fourth is equality of raw and groupoid partition sums for finite actions.

The fifth is the one-object recovery theorem.

The sixth is invariance under relabeling and equivalent groupoid presentation.

Formalization serves several purposes.

It separates theorem from interpretation.

It detects hidden source-target errors.

It clarifies assumptions.

It permits exact reuse of definitions.

It provides a verified reference against which numerical code can be tested.

The full physical theory need not be formalized before computation begins.

A minimal verified core is sufficient.

The recommended structure is:

Definitions:
finite oriented complex,
finite groupoid,
configuration,
gauge transformation,
path transport,
holonomy,
action.

Lemmas:
orientation reversal,
path composition,
intermediate gauge cancellation,
basepoint conjugation.

Theorems:
holonomy covariance,
class-function invariance,
action descent,
orbit-stabilizer partition identity,
one-object lattice-gauge recovery.

Corollaries:
gauge invariance of Wilson loops,
presentation independence of defect-sector minima.

The computational code should generate test cases that can be checked against the formal model.

For very small systems, the proof assistant or verified evaluator may calculate exact orbit and action results.

These serve as golden test vectors.

Higher-categorical formalization can follow later.

Homotopy Type Theory is especially relevant for:

identity paths;

higher equivalences;

truncations;

and univalence.

However, the first formal implementation may use ordinary finite groupoids in a conventional dependent type theory.

The mathematical content should be proven at the level required by the model rather than invoking the full HoTT framework ceremonially.

11.10 Reproducibility and Open Computational Record

The computational program should be designed for independent replication from the beginning.

Every result must be associated with:

source code;

version identifier;

input files;

random seeds;

complete parameters;

hardware and software environment;

raw output;

analysis scripts;

and a human-readable run description.

The public record should distinguish:

exploratory runs;

failed runs;

confirmatory runs;

and final reported runs.

Negative results should not be discarded silently.

They reveal:

unstable parameter regions;

failed defect searches;

nonselection of sectors;

and algorithmic limitations.

A standard directory structure may include:

models/
definitions of groups, groupoids, complexes, and actions;

runs/
complete run configurations and seeds;

raw/
unmodified outputs;

processed/
derived tables and observables;

figures/
generated plots;

proofs/
formal verification files;

tests/
unit and invariance tests;

docs/
model descriptions and replication instructions.

Each run should have a manifest containing:

run identifier;

timestamp;

code commit;

host;

model name;

complex;

groupoid;

couplings;

boundary conditions;

algorithm;

seed;

duration;

and output checksum.

Random-seed control is necessary but not sufficient.

Reproducibility should also include independent implementations of key calculations.

For example:

one orbit enumerator using direct group action;

another using gauge fixing;

one partition calculation using raw sums;

another using orbit weights.

Agreement reduces the chance of common coding assumptions contaminating the result.

Parameter sweeps should be declared before confirmatory analysis.

If hundreds of combinations are searched and only the most dramatic case reported, the apparent significance is inflated.

The search procedure should report:

total parameter combinations;

selection criteria;

number of positive cases;

and independent confirmation.

Error bars should include:

sampling uncertainty;

finite-size uncertainty;

fit-model uncertainty;

and normalization sensitivity.

A single numerical value without these components should not be treated as a prediction.

The code should contain automated tests for:

groupoid axioms;

complex consistency;

gauge invariance;

orbit-stabilizer identity;

orientation reversal;

partition-sum equality;

and known benchmark values.

A release should not be tagged as scientifically usable until all mandatory tests pass.

The data formats should be documented.

Group and groupoid definitions should be portable.

Complexes should use standard graph or cell-complex representations where possible.

Results should be exportable to ordinary tables so that they can be analyzed independently of GenesisBoard.

Computational claims should be reproducible on modest hardware for the smallest benchmark models.

Large-scale runs may require a cluster, but the logical core should not depend on inaccessible resources.

The purpose of open computation is not merely transparency.

It also allows the wider mathematical and physics communities to test alternative:

groupoids;

actions;

boundary conditions;

and scaling methods.

A ToE candidate must invite attempts at refutation.

11.11 Decision Criteria for the Computational Program

The computational stage should be governed by explicit decision points.

The first decision concerns mathematical correctness.

Proceed only if:

gauge invariance passes;

orbit counting is correct;

and the one-object model reproduces known lattice results.

The second decision concerns defect matter.

Proceed to mass interpretation only if:

defect labels are gauge invariant;

minimum configurations are localized;

and stability survives perturbation.

The third decision concerns mass.

Proceed to continuum claims only if:

gauge-invariant correlations yield a reproducible gap;

the gap survives finite-size analysis;

and mass ratios stabilize.

The fourth decision concerns gauge selection.

Proceed to Standard Model-like candidate spaces only if:

finite sector selection survives reasonable priors and normalizations;

and selection is not determined by trivial group size.

The fifth decision concerns geometry.

Proceed to gravity claims only if:

the relational distance is gauge invariant;

multiple dimension estimators agree;

and defect energy produces universal geometric response.

The sixth decision concerns thermodynamic gravity.

Proceed to Einstein-like derivation only if:

subregion entropy has an area-dominated regime;

an effective temperature is defined;

and energy flux correlates with geometric focusing.

The seventh decision concerns higher structure.

Add 2-morphisms only when they yield:

new invariant sectors;

required coherence;

or quantitative improvement.

Higher categories should not be added merely because the foundational language permits them.

The eighth decision concerns falsification.

Abandon or revise a model variant when:

its principal effect disappears under controls;

its results depend on presentation;

its continuum tendency fails;

or its predictions are entirely parameter inserted.

These decision points protect the research program from indefinite expansion without empirical or mathematical gain.

The aim is not to preserve every conjecture.

The aim is to identify which parts of Univalent Relational Dynamics survive construction.

11.12 From Simulation to Physical Theory

A computational model becomes physically significant only when its quantities can be connected to established observables or experimental constraints.

The first bridge is internal consistency with known gauge theory.

The second is recovery of dimensionless spectra and scaling behavior.

The third is identification of units.

A finite action is often dimensionless in lattice units.

To compare with physical mass, distance, or coupling, one quantity must set the scale.

This may be:

a correlation length;

a known mass;

a critical coupling;

or an emergent geometric unit.

A true derivation should minimize the number of externally calibrated quantities.

The fourth bridge is continuum effective field theory.

Coarse-grain the finite model and identify the operators appearing in the long-distance action.

Determine whether it approaches:

Yang-Mills theory;

scalar gauge theory;

topological field theory;

gravity;

or another known universality class.

The fifth bridge is phenomenology.

If the model predicts:

a mass ratio;

a restricted charge;

a new topological sector;

a modified correlation law;

or a scale-dependent dimension,

compare it with current observations and experiments.

The sixth bridge is falsifiability.

A prediction must exclude some possible observation.

For example:

a sector-selection rule may forbid a class of gauge groups;

a defect spectrum may require a mass ordering;

an entropy correction may modify black-hole thermodynamics;

a dimensional-flow law may alter high-energy propagation;

or a coherence-induced interaction may produce a measurable deviation.

The simulation should not be presented as reality merely because it produces visually complex patterns.

Many simple computational systems generate intricate structures.

Physical relevance requires:

invariance;

recovery;

universality;

quantitative constraint;

and observational connection.

The role of GenesisBoard is therefore precise.

It is a laboratory for determining whether the equivalence-first principles can support a coherent physical theory.

It can demonstrate:

existence;

stability;

selection;

scaling;

and failure.

It cannot by itself establish that the universe is fundamentally a GenesisBoard computation.

The strongest defensible outcome of this section would be:

a fully reproducible finite implementation of Univalent Relational Gauge Dynamics that exactly recovers known gauge models, produces gauge-invariant defect sectors, calculates their quantum spectrum, and identifies at least one robust non-inserted result.

Such a result would move the program from conceptual unification to constructive mathematical physics.

The final section defines how that achievement should be evaluated, what would falsify the framework, and what additional steps remain before it could be regarded as a candidate Theory of Everything.

12. Predictions, Falsification, and the Path Toward a Theory of Everything

12.1 Minimal Success

The present theory should be evaluated first by the standards appropriate to a constructive finite model. Its earliest success does not require the immediate derivation of the Standard Model, general relativity, or all observed particle parameters. It requires the demonstration that the equivalence-first ontology can be converted into a mathematically complete, internally consistent, gauge-invariant, and computationally reproducible field theory.

The minimum successful result consists of several linked achievements.

First, the theory must define its mathematical objects without ambiguity.

The finite oriented 2-complex K must be specified through:

vertices;

oriented edges;

faces;

incidence relations;

orientation conventions;

boundary conditions;

and any temporal or geometric structure used in the calculation.

The physical groupoid G must be specified through:

objects;

morphisms;

sources and targets;

identity morphisms;

inverses;

composition;

automorphism groups;

and admissibility conditions.

A field configuration Phi must be defined as an assignment of:

local identity objects A_v;

transport morphisms U_e;

matter variables psi_v where present;

and all additional sector or boundary data.

The local gauge action must be defined exactly.

The action S_URD must be given as an explicit calculable functional.

The observables must have defined domains, codomains, and transformation behavior.

A model containing undefined phrases such as coherence, identity cost, or admissible equivalence without mathematical implementation has not yet reached minimal success.

Second, the groupoid and complex constructions must satisfy their own axioms.

The composition of morphisms must be associative.

Identity arrows must act neutrally.

Every reversible morphism must possess the stated inverse.

Path composition must agree with transport composition.

Face boundaries must close.

Orientation reversal must be implemented consistently.

The computational representation must not merely resemble a groupoid or complex. It must satisfy the exact structural laws.

Third, the gauge transformation must define an action on the configuration space.

For each family:

g = {g_v},

with:

g_v in Aut(A_v),

the transformed edge transport must be:

U_e' = g_t(e) o U_e o g_s(e)^-1.

The matter field must transform through the appropriate representation.

Successive gauge transformations must compose correctly.

The identity gauge transformation must leave the configuration unchanged.

Every gauge transformation must be invertible.

Fourth, the complete action must be gauge invariant.

For every allowed configuration Phi and every admissible local gauge transformation g:

S_URD(g · Phi) = S_URD(Phi).

This must be shown analytically and verified computationally.

Each action component must satisfy its own invariance requirement.

A cancellation in the total action that hides a nonphysical gauge-dependent term is not sufficient unless the cancellation is exact and structurally justified.

The face term must depend only on conjugacy-invariant holonomy data.

The matter term must use covariant differences and invariant norms.

The identity cost must be constant on physically equivalent object presentations.

The boundary terms must transform according to the declared boundary convention.

The defect classifier must remain unchanged under gauge transformation.

Fifth, the observables must descend to the physical moduli structure.

A claimed physical quantity cannot depend on:

local frame;

vertex labeling;

edge naming;

face basepoint;

group-element representation;

or another arbitrary presentation choice.

Wilson loops, defect charges, partition functions, action gaps, mass estimators, and relational distances must all pass this requirement.

Gauge-dependent quantities may be used internally for computation, but every physical conclusion must be expressible through a descending observable.

Sixth, the model must recover ordinary lattice gauge theory in the one-object limit.

Let the groupoid contain one object A with:

Aut(A) = G.

Every edge transport then belongs to G.

The local gauge transformation becomes the standard lattice transformation.

The face holonomy becomes the plaquette product.

With a Wilson-type class function, the face-coherence action must reduce to the Wilson lattice action up to stated normalization.

If matter is included, the edge-covariant difference must reduce to the standard lattice gauge-matter kinetic term.

This recovery must be exact at the finite level.

The theory cannot claim to generalize gauge physics if it fails its most immediate special case.

Seventh, at least one nontrivial finite model must be solved completely.

Complete solution means that the calculation obtains:

all raw configurations;

all gauge orbits;

all stabilizers;

the action spectrum;

the exact partition function;

and selected observable expectations.

A one-face Z_2 model may be used as an initial test, but the strongest minimal demonstration should involve enough structure to contain interacting faces, nontrivial topology, or a non-Abelian group.

The exact raw sum and the automorphism-weighted orbit sum must agree.

This calculation establishes that the moduli-groupoid quantization is not merely formal.

Eighth, the theory must produce at least one gauge-invariant nontrivial sector.

This may be:

a global holonomy sector;

a boundary-forced flux sector;

a matter-winding sector;

a domain wall;

or another obstruction class.

The sector must be distinguishable from the vacuum by a physical invariant.

It must not be removable by a gauge transformation.

It need not yet be a realistic particle.

Its existence shows that the model supports physical distinctions beyond gauge presentation.

Ninth, the theory must demonstrate exact reproducibility.

The published result should include:

source code;

group and groupoid definitions;

complex definition;

parameters;

random seeds where applicable;

raw output;

orbit data;

analysis scripts;

and tests.

An independent investigator should be able to reconstruct the exact finite result.

The minimum claim supported by these achievements is:

Univalent Relational Dynamics can be formulated as a constructive finite gauge theory whose physical configurations are moduli-groupoid objects and whose one-object sector reproduces ordinary lattice gauge dynamics.

This is not a Theory of Everything.

It is nevertheless a decisive transition.

The program would have moved from a philosophical and ontological proposal to a functioning mathematical theory.

The importance of this threshold should not be underestimated.

Many broad foundational programs fail before reaching it because their central concepts cannot be converted into consistent variables, actions, and observables.

A successful finite construction demonstrates that:

identity objects can be assigned locally;

equivalences can act as gauge transformations;

transport can generate holonomy;

coherence cost can define an action;

and quantization can be performed over physical equivalence classes.

The minimal result also provides a foundation for failure.

Once the theory is explicit, its extensions can be shown to work or not work.

Without this constructive core, every later claim about matter, geometry, or gravity remains too flexible.

12.2 Strong Success

A stronger success requires the model to produce physical structure that was not defined into the starting data.

Recovery of lattice gauge theory is necessary, but it does not establish new physics. The model becomes substantively original when its enlarged equivalence structure generates a robust result that a fixed-group theory would have treated as an input.

Several levels of strong success are possible.

The first is a stable defect sector with a nonzero invariant coherence gap.

The model should identify a defect class d such that:

the class is gauge invariant;

the class cannot be continuously removed under the allowed dynamics;

the minimum-action configuration is localized;

the excess action remains finite as the system grows;

and the defect survives small admissible perturbations.

Define:

Delta S_d
= S_min(d) - S_min(0).

If Delta S_d is positive and robust, the model has generated a cost associated with persistent nontrivial identity.

This is the first quantitative realization of a particle-like coherence defect.

The second level is a transfer-matrix or correlation-function mass.

A gauge-invariant operator O_d should couple to the defect sector.

The Euclidean correlator:

C_d(t)
= <O_d(t) O_d(0)^dagger>

should exhibit a stable asymptotic decay rate.

The extracted gap:

Delta_d

should agree, within the expected finite-volume and normalization effects, with the lowest energy in the corresponding transfer-matrix sector.

The result should survive:

different operators;

different gauge choices;

different lattice sizes;

and different fitting procedures.

This supports mass as a physical spectral property rather than an arbitrary action label.

The third level is transportability.

The defect should be movable across the complex without losing its invariant identity.

A sequence of configurations should carry the defect from one region to another while preserving:

its obstruction class;

its automorphism representation;

and its minimum-energy structure.

If an effective momentum can be defined, the excitation should possess a dispersion relation.

An approximately relativistic form:

E(p)^2
approximately equals
m^2 c^4 + p^2 c^2

would provide strong evidence that the defect behaves as a particle rather than as a static lattice imperfection.

The fourth level is charge as an emergent automorphism representation.

The defect-state space should transform under:

rho_d : Aut(A) -> GL(V_d).

The representation class must not be assigned externally as a label with no dynamical origin.

It should arise from the action of the selected local self-equivalence structure on the physical defect sector.

The charge should be detectable through:

boundary symmetry;

loop holonomy;

fusion behavior;

or a Gauss-law-like constraint.

Composite defects should transform according to representation products.

Opposite or dual charge sectors should permit neutral combinations.

The fifth level is a quantized defect spectrum.

The model may allow only certain obstruction and representation classes.

If local coherence, topology, and admissibility eliminate other possibilities, the allowed charges are derived rather than chosen.

A discrete set of stable sectors is more explanatory than an arbitrary continuum of labels.

The sixth level is a robust mass ratio.

Suppose two defect sectors d_1 and d_2 have gaps:

Delta_1 and Delta_2.

If the ratio:

Delta_1 / Delta_2

approaches a stable value under:

increasing size;

refinement;

reasonable changes of microscopic action;

and common scale setting,

the model has generated a dimensionless prediction.

A mass ratio is especially valuable because it does not depend on the arbitrary overall choice of lattice unit.

The seventh level is dynamical automorphism-sector selection.

Let the theory contain candidate identity components C_i with automorphism groups G_i.

Using common action rules and stated sector priors, calculate:

Z_i

and:

F_i = -k_B T log Z_i.

A strong result occurs when one sector or restricted class of sectors dominates over a broad parameter region without sector-specific tuning.

The preference should survive:

increasing system size;

alternative reasonable normalizations;

automorphism weighting;

and the inclusion of basic matter or defect effects.

This would show that gauge structure can be selected dynamically rather than merely postulated.

The eighth level is matter-and-gauge coselection.

A more ambitious result would select:

an automorphism group;

a set of stable representations;

and a symmetry-breaking pattern

through one coupled partition function.

The gauge structure and matter content would then be mutually consistent outputs.

This would begin to address the deeper ToE requirement that forces and particles emerge together.

The ninth level is emergent geometry.

The gauge-invariant transport cost should define a stable relational metric or pseudometric.

Multiple dimension estimators should agree over a growing scale interval.

Correlation propagation should follow the same geometry.

Defects should distort relational distances in a systematic manner.

The resulting dimension should not be merely inherited from the initial lattice.

A non-inserted effective dimension would be one of the strongest results available to the framework.

The tenth level is universal geometric response.

Construct two defect species with different internal charge representations but equal total energy.

If both produce the same large-scale relational curvature or distance distortion, the model exhibits a primitive equivalence principle.

This would support the interpretation of gravity as the universal response of coherence geometry to energy.

The eleventh level is an entropy-area relation.

For subregions with fixed gauge-invariant boundary data, calculate the automorphism-corrected entropy.

If the leading physically relevant contribution scales with boundary area rather than volume in a controlled regime, the model gains a concrete bridge toward horizon thermodynamics.

The twelfth level is a thermodynamic focusing relation.

A controlled change in energy flux across an effective causal boundary should produce:

a change in equivalence entropy;

a change in boundary area;

and a change in geodesic or path focusing.

A stable relationship among these quantities would move the gravity proposal beyond analogy.

The thirteenth level is a new selection rule or observable deviation.

The model might predict:

forbidden automorphism groups;

restricted defect charges;

a universal mass relation;

a topological correction to amplitudes;

a modified entropy term;

a scale-dependent dimension;

or a deviation from conventional gauge behavior.

The prediction must follow from the model before comparison with data.

It must exclude some possible result.

A strong success should be described in proportion to what has actually been achieved.

For example:

A gauge-invariant stable defect with a finite lattice gap supports the claim of emergent defect matter in the finite model.

It does not yet establish a fundamental particle.

A preferred finite automorphism sector supports dynamical gauge selection in a toy ensemble.

It does not yet derive the Standard Model.

A robust relational dimension supports emergent pregeometry.

It does not yet derive Lorentzian spacetime.

An area-law entropy supports the thermodynamic bridge.

It does not yet prove Einstein's field equations.

Careful staging strengthens rather than weakens the theory.

The strongest early publication would combine three accomplishments:

exact recovery of lattice gauge theory;

one robust non-inserted defect or sector result;

and a complete reproducible computational implementation.

That combination would establish Univalent Relational Gauge Dynamics as an original candidate field framework rather than as a reinterpretation of known mathematics.

12.3 Falsification Criteria

A foundational theory must specify not only what would support it but what would count against it.

The breadth of Univalent Relational Dynamics creates a special danger. Almost any physical structure can be described loosely using words such as relation, equivalence, coherence, or obstruction. Without explicit rejection criteria, the program could absorb every result without risk.

The constructive model prevents this by turning the claims into exact tests.

The first falsification criterion concerns definability.

If no precise admissibility selector can be constructed, the theory lacks a physical equivalence relation.

The mathematical category may contain many isomorphisms, but the theory must determine which preserve physical identity.

If admissibility remains an informal judgment applied differently in each example, the framework is not predictive.

The second criterion concerns closure.

The selected physical equivalences must be closed under identity, inverse, and composition.

If admissible equivalence fails these laws without a precisely defined weaker structure replacing them, the groupoid interpretation is invalid.

The third criterion concerns gauge invariance.

If:

S_URD(g · Phi) != S_URD(Phi)

for an admissible local change of presentation, the action depends on description rather than physical identity.

The same applies to every proposed observable.

A gauge-dependent mass, defect label, or distance is not physical within the theory.

The fourth criterion concerns one-object recovery.

If the fixed-object restriction does not reproduce ordinary lattice gauge structure, the proposed generalization fails its most immediate consistency requirement.

Incorrect orientation order, basepoint dependence, or noncovariant matter coupling would be direct mathematical failures.

The fifth criterion concerns overcounting.

If the raw configuration sum and the automorphism-weighted orbit sum disagree in a finite model after all conventions are matched, the quantization measure is incorrect.

If redundant equivalent object presentations change the physical partition function, the theory is counting description as reality.

The sixth criterion concerns defect removability.

If every candidate defect can be continuously deformed or gauge transformed into the vacuum without changing physical boundary data, the model has not generated stable matter.

Transient curvature fluctuations should not be promoted to particles.

The seventh criterion concerns localization.

If the minimum nontrivial sector spreads throughout the entire system and its radius grows with system size, it is not a localized particle-like excitation.

It may still describe a global flux or alternate vacuum.

The eighth criterion concerns invariant mass.

If the extracted gap depends on:

gauge representative;

face basepoint;

arbitrary lattice labeling;

or equivalent identity presentation,

mass as equivalence inertia fails.

The ninth criterion concerns scaling.

If every finite gap vanishes under the relevant large-volume or continuum extrapolation, the model has not generated massive continuum excitations.

If every defect energy diverges uncontrollably under refinement, the construction also fails as a physical continuum theory.

The tenth criterion concerns transportability.

If a localized defect cannot move without losing its physical identity or encountering an infinite lattice artifact, it cannot model a freely propagating particle.

The eleventh criterion concerns dispersion.

If a theory claims fundamental relativistic particles but the defect excitations possess no approximate relativistic dispersion in the intended low-energy regime, the identification is unsupported.

The model may remain useful as a nonrelativistic or condensed-matter analogue, but not as a fundamental particle theory.

The twelfth criterion concerns charge.

If the defect representation changes under gauge choice or motion, it is not a stable charge.

If composite defects violate the representation-fusion rules or total charge is not conserved under allowed dynamics, charge as automorphism response fails in that model.

The thirteenth criterion concerns arbitrary gauge-sector selection.

If the preferred automorphism group is determined entirely by:

an arbitrary prior;

group order;

a manually lowered baseline cost;

or a chosen representation favoring the desired answer,

the theory has not derived gauge structure.

The fourteenth criterion concerns prior instability.

If small reasonable changes in the sector measure reverse the selected group without an independent physical reason, the result is not robust.

The fifteenth criterion concerns candidate-space dependence.

If the selected answer changes whenever one adds another plausible group or identity object, the theory has not established a stable selection law.

The sixteenth criterion concerns continuum viability.

A finite sector that has no unitary, local, stable, or renormalizable continuum limit cannot serve as fundamental physics.

The seventeenth criterion concerns realistic matter.

A gauge-sector result that cannot support:

chiral fermions;

anomaly cancellation;

stable representations;

or observed interaction patterns

cannot become a Standard Model replacement.

The eighteenth criterion concerns geometry.

If the proposed relational distance changes under gauge transformation, it fails immediately.

If different low-energy probes infer irreconcilable geometries, the theory has not produced universal spacetime.

If the effective dimension simply matches the chosen lattice dimension under every condition, dimension has not emerged.

The nineteenth criterion concerns curvature.

If holonomy cost has no stable relation to relational-distance distortion, geodesic deviation, or frame transport, internal gauge curvature has not become gravitational curvature.

The analogy should then be abandoned or restricted.

The twentieth criterion concerns equivalence entropy.

If gauge-related presentations are counted as distinct thermodynamic states, the entropy is wrong.

If the entropy result depends on arbitrary presentation multiplicity, it is not physical.

The twenty-first criterion concerns area scaling.

If the relevant subregion entropy remains dominantly volumetric in every candidate gravitational regime, the proposed horizon bridge is unsupported.

The twenty-second criterion concerns thermodynamic gravity.

If no universal relationship emerges among:

energy flux;

entropy change;

boundary geometry;

and focusing,

the thermodynamic derivation does not follow from URD.

Writing delta Q = T delta S is not itself evidence.

The twenty-third criterion concerns universality of gravity.

If equal-energy defects with different internal charges produce different large-scale free-fall or geometric response, the model violates the required equivalence behavior.

The twenty-fourth criterion concerns measurement.

If the truncation map depends on gauge presentation, it cannot represent physical observation.

If the model cannot produce stable records or suppress interference through explicit environmental interaction, measurement as truncation remains purely verbal.

The twenty-fifth criterion concerns probabilities.

If the theory claims to replace quantum mechanics but cannot derive or reproduce the Born rule, the claim is incomplete.

The framework may still organize amplitudes and decoherence, but it has not reconstructed quantum prediction.

The twenty-sixth criterion concerns unitarity.

If the transfer matrix is not positive where positivity is required or if the Lorentzian evolution fails to preserve probability, the quantum theory is inconsistent.

The twenty-seventh criterion concerns causality.

If the continuum model permits uncontrolled acausal signaling or lacks a stable causal structure in a regime claimed to reproduce relativity, the ToE interpretation fails.

The twenty-eighth criterion concerns parameter proliferation.

If every observed mass, coupling, charge, and symmetry requires a separate adjustable coherence coefficient, the model has not reduced explanatory input.

It may be a reformulation with more parameters than the theories it replaces.

The twenty-ninth criterion concerns nonuniqueness.

If infinitely many unrelated coherence actions reproduce the same conceptual dictionary and the framework provides no principle for selecting among them, the theory lacks determinate dynamics.

The thirtieth criterion concerns empirical indistinguishability.

If URD reproduces existing physics exactly but generates no new constraint, relation, or observable, it remains an interpretation rather than an independently testable theory.

Interpretive value is legitimate, but it should not be presented as a predictive ToE.

The thirty-first criterion concerns computational irreproducibility.

If the central result depends on unpublished code, undocumented filtering, selected seeds, or unrecoverable run conditions, it cannot support a scientific claim.

The thirty-second criterion concerns post hoc selection.

If a broad parameter search is performed and only the most favorable result is reported, apparent prediction is weakened.

The search space, selection procedure, and independent confirmation must be disclosed.

The thirty-third criterion concerns higher-categorical ornamentation.

If adding 2-morphisms, HoTT language, or univalence produces no new definition, theorem, calculation, or constraint, the higher structure is ornamental.

It should be removed from that model until it becomes operationally necessary.

The thirty-fourth criterion concerns explanatory circularity.

If the model assumes the same structure it claims to derive, the derivation is circular.

Examples include:

choosing the Standard Model group and claiming gauge selection;

inserting particle masses and claiming mass emergence;

choosing a four-dimensional Lorentzian lattice and claiming spacetime emergence;

or imposing an area law and claiming gravitational entropy derivation.

The thirty-fifth criterion concerns total-theory compatibility.

A successful gauge sector, gravity sector, and quantum sector must coexist within one mathematical framework.

If they require mutually incompatible definitions of equivalence, time, probability, or geometry, the program does not form a unified theory.

These criteria should be treated as active tests, not as ceremonial disclaimers.

The project should publish negative results when a proposed model fails.

A failure of one action does not necessarily falsify the entire equivalence-first ontology.

It falsifies that particular implementation or claimed derivation.

However, repeated failure across broad reasonable model classes would weaken the foundational proposal itself.

For example:

If no gauge-invariant relational cost produces universal geometry across many constructions, the claim that spacetime emerges from equivalence cost becomes doubtful.

If no stable defect masses arise without inserted mass terms, mass as equivalence inertia becomes unsupported.

If automorphism-sector dominance is always prior dependent, gauge selection through coherence may fail as a foundational mechanism.

The theory advances by narrowing what remains possible.

12.4 Relation to a Theory of Everything

A Theory of Everything is not merely a vocabulary broad enough to describe every area of physics.

It is a mathematical structure from which the established fundamental theories arise as controlled limits and from which at least some of their apparently independent inputs are derived.

Univalent Relational Dynamics presently offers a candidate organizing principle:

physical identity is constituted by admissible equivalence;

dynamics governs the cost and obstruction of coherent transport;

matter corresponds to persistent nontrivial sectors;

and geometry arises from the relational organization of transport.

This principle can unify the interpretation of many structures.

That conceptual unity is necessary for a ToE candidate but not sufficient.

A completed theory must recover at least four major domains.

The first is quantum theory.

The model must reconstruct:

complex amplitudes;

composition of processes;

Hilbert-space or equivalent state structure;

unitarity;

tensor composition;

superposition;

interference;

entanglement;

measurement;

and the Born rule.

The moduli-groupoid path sum provides a candidate amplitude structure.

Measurement as truncation provides a possible account of classical outcomes.

Neither is complete without probability derivation and a demonstrated quantum-state formalism.

The second domain is gauge quantum field theory.

The theory must recover:

local gauge symmetry;

Yang-Mills dynamics;

matter representations;

fermions;

chirality;

gauge fixing or quotient quantization;

renormalization;

confinement where applicable;

and scattering amplitudes.

The one-object recovery addresses only the finite lattice beginning of this requirement.

The third domain is general relativity.

The theory must recover:

Lorentzian 3+1-dimensional spacetime;

local causal structure;

the equivalence principle;

metric or frame geometry;

curvature;

energy-momentum conservation;

Einstein-like field equations;

and tested weak- and strong-field limits.

Relational distance and thermodynamic gravity currently provide a research pathway, not a completed derivation.

The fourth domain is thermodynamics and statistical mechanics.

The theory must explain:

physical microstates;

coarse macrostates;

entropy;

irreversibility;

equilibrium;

temperature;

and the arrow of time.

Equivalence-fiber entropy offers a unifying formulation, but the emergence of the second law still requires statistical and boundary-condition arguments.

A ToE must also explain the interface among these domains.

Quantum matter must source gravity.

Gravity must affect quantum propagation.

Gauge charges must coexist with spacetime geometry.

Thermodynamic entropy must agree with quantum and gravitational entropy.

Measurement and causal structure must be compatible.

It is not enough to construct four parallel models with related terminology.

They must arise from one action, one state or process structure, and one definition of physical equivalence.

The present paper proposes the finite kernel:

local identity objects;

admissible transport;

coherence action;

moduli-groupoid quantization;

defect sectors;

and relational geometry.

The ToE question is whether this kernel can generate all required effective structures without contradiction or uncontrolled additional assumptions.

Several specific Standard Model achievements remain.

The theory must derive or constrain the gauge group.

A realistic target is not merely the Lie algebra:

$su(3) + su(2) + u(1)$,

but the correct global group structure, including any discrete quotient relevant to representations.

The theory must derive the observed matter representations.

These include:

quark color triplets;

weak doublets;

singlets;

hypercharge assignments;

and the separation of left- and right-handed fields.

The theory must explain anomaly cancellation.

The allowed representations must satisfy local and global consistency conditions.

The theory must explain three generations.

A category of defects may possess repeated sectors, but repetition alone does not explain why there are exactly three.

The theory must derive or constrain the Higgs sector.

It must explain the vacuum symmetry reduction and the resulting gauge-boson masses.

The theory must address Yukawa couplings.

These determine fermion masses and mixing.

A ToE should reduce the apparent arbitrariness of these matrices.

The theory must account for neutrino masses and mixing.

It must determine whether neutrinos are Dirac, Majorana, or another relational structure.

The theory must reproduce strong-interaction behavior, including asymptotic freedom and confinement.

The theory must explain or accommodate CP violation and the strong CP problem.

The gravitational achievements are equally demanding.

The model must explain why the effective dimension is 3+1.

It must explain why the signature is Lorentzian.

It must identify the origin and value of Newton's constant.

It must address the cosmological constant or vacuum-energy problem.

It must describe black-hole entropy and evaporation consistently.

It must provide a quantum description of gravitational degrees of freedom.

It must recover cosmological expansion and structure formation.

The ToE program should also address phenomena not explained completely by the current Standard Model plus general relativity.

These include:

dark matter;

dark energy;

matter-antimatter asymmetry;

inflation or an alternative early-universe mechanism;

quantum gravity;

black-hole information;

and the nature of measurement.

Univalent Relational Dynamics need not solve every puzzle in its first technical realization.

But a mature ToE must show where each phenomenon fits and which are derived, emergent, or absent.

The number of free inputs provides another measure of progress.

The current fundamental theories contain:

gauge groups;

representations;

couplings;

masses;

mixing parameters;

vacuum parameters;

and cosmological quantities.

A ToE should explain why these inputs take their observed values or at least derive relations among them.

If URD introduces an independent coherence parameter for each existing constant, no explanatory compression has occurred.

The framework becomes stronger when a small set of principles determines many outputs.

Possible deep inputs might include:

the class of admissible identity structures;

the universal form of the coherence action;

the quantum measure;

and boundary conditions.

Everything else should ideally emerge.

This ideal may not be fully attainable.

Some boundary or initial conditions may remain contingent.

The theory should distinguish law from state.

A ToE may determine the space of possible universes without selecting one unique cosmological history.

However, observed particle physics should not be relegated entirely to arbitrary initial conditions if the theory claims fundamental explanation.

The status of the present program can therefore be expressed on a ladder.

Level 1 is conceptual unification.

The equivalence-first ontology reaches this level.

Level 2 is constructive finite theory.

The present paper aims to reach this level.

Level 3 is recovery of established gauge and quantum models.

The one-object lattice sector begins this work.

Level 4 is non-inserted matter, symmetry, or geometry.

This requires successful defect, selection, or dimension results.

Level 5 is controlled continuum field theory.

The finite model must possess a universal scaling limit.

Level 6 is unified quantum gauge-gravity dynamics.

Internal and geometric sectors must interact consistently.

Level 7 is derivation of realistic particle and cosmological structure.

Level 8 is novel quantitative prediction.

Level 9 is empirical confirmation.

Only the later levels justify a strong ToE claim.

The phrase candidate Theory of Everything may be used at an intermediate stage if clearly qualified.

A candidate ToE architecture is a framework specifying how all fundamental sectors could arise.

A candidate ToE model is a single mathematically defined theory that recovers substantial parts of known physics.

A completed ToE is a theory that successfully reproduces the established domains, explains major inputs, and survives empirical tests.

The present work aims to convert the architecture into the first candidate model.

It should not claim completion.

12.5 Research Sequence

The order in which the program is developed matters.

Attempting to derive all physics simultaneously would make failure impossible to diagnose. Each stage should introduce one new layer while preserving the verified results of earlier stages.

The recommended sequence begins with finite exact gauge dynamics.

Stage 1 is the one-object benchmark.

Choose small finite groups.

Implement:

edge transport;

gauge transformations;

face holonomy;

Wilson-type action;

orbit counting;

stabilizers;

and exact partition functions.

Recover known lattice results.

This stage validates the mathematical and computational foundation.

Stage 2 introduces physical groupoids with multiple equivalent object presentations.

The purpose is to test whether:

different local representatives;

connecting isomorphisms;

and automorphism-weighted counting

leave physical observables unchanged.

This stage verifies presentation independence beyond a one-object group.

Stage 3 introduces nontrivial topology and defect sectors.

Use:

periodic complexes;

punctures;

boundary flux;

and matter winding.

Classify global holonomy and obstruction sectors.

Calculate their minimum actions.

Determine which are localized and stable.

Stage 4 introduces Euclidean time and transfer matrices.

Construct temporal slabs.

Calculate:

ground states;

sector spectra;

correlation functions;

and finite mass gaps.

This stage converts static coherence costs into quantum excitation energies.

Stage 5 tests defect charge.

Determine the automorphism representation of each stable sector.

Construct charge projectors.

Test fusion, duality, and conservation.

At the completion of this stage, the theory may support a finite model of emergent matter with mass and charge.

Stage 6 introduces heterogeneous identity sectors.

Compare candidate finite groups or groupoids under common action rules.

Calculate restricted partition functions and free energies.

Test whether any automorphism sector is selected robustly.

Stage 7 introduces sector interfaces and noninvertible transitions.

Define domain walls, nucleation, and identity-changing processes.

This requires a larger category beyond the reversible groupoid.

Study real-time or Euclidean tunneling between identity sectors.

Stage 8 introduces higher morphisms.

Use finite 2-groups, crossed modules, or 2-groupoids.

Add surface transport and gauge-of-gauge structure.

Determine whether higher coherence produces:

new charge sectors;

new anomalies;

or new defect stability.

Stage 9 develops coarse-graining and renormalization.

Construct blocking maps for:

identity objects;

edge transports;

faces;

and defect variables.

Determine which operators are generated.

Search for fixed points and universal scaling trajectories.

This stage determines whether the finite model can become a continuum field theory.

Stage 10 extends to compact Lie groups and realistic gauge structures.

Use the insights from finite-group selection to compare:

unitary;

special unitary;

orthogonal;

symplectic;

exceptional;

and product groups.

Impose representation and anomaly constraints.

This stage should not begin until the finite selection mechanism is understood.

Stage 11 introduces fermions and chirality.

Construct:

spin or frame structure;

discrete Dirac operators;

Grassmann variables or equivalent finite formulations;

and chiral representation constraints.

Test anomaly cancellation.

This is indispensable for any connection to the Standard Model.

Stage 12 investigates generation structure.

Search for repeated defect families arising from:

topology;

higher homotopy;

categorical grading;

or representation multiplicity.

The target is not merely to fit three generations but to derive why three stable copies occur.

Stage 13 studies symmetry breaking and the Higgs mechanism.

Determine whether a selected identity sector supports a vacuum with residual automorphism group matching observed low-energy symmetry.

Calculate gauge-boson and matter excitation gaps.

Stage 14 develops relational geometry.

Define gauge-invariant transport costs.

Calculate effective distances, dimensions, and curvature.

Test universality across probes.

Determine whether defects backreact on the same geometry.

Stage 15 introduces frame and coframe identity data.

Connect internal transport to tangent or spacetime structure.

Study torsion, nonmetricity, and metric compatibility.

Search for a Lorentzian phase.

Stage 16 develops causal and temporal emergence.

Distinguish reversible equivalence from directed physical process.

Construct causal cones and effective propagation speed.

Determine whether one temporal and three spatial dimensions are dynamically preferred.

Stage 17 develops equivalence entropy.

Calculate subregion state counts with gauge and automorphism correction.

Study area, volume, and topological scaling.

Connect measurement truncation, decoherence, and thermodynamic coarse-graining.

Stage 18 tests the thermodynamic gravity bridge.

Define local causal boundaries.

Calculate effective temperature, energy flux, entropy change, and geometric focusing.

Determine whether an Einstein-like relation follows.

Stage 19 constructs unified gauge-gravity dynamics.

The same identity structure should support:

internal automorphisms;

frame automorphisms;

matter defects;

and geometry.

The interactions must be derived from one action rather than appended as independent sectors.

Stage 20 develops phenomenology.

Calculate dimensionless predictions:

mass ratios;

coupling relations;

charge restrictions;

entropy corrections;

propagation deviations;

or cosmological signatures.

Compare them with observation.

Each stage must include a stopping rule.

Do not proceed merely because the next mathematical language is available.

Proceed when the preceding stage satisfies its success criteria.

For example:

Do not interpret action gaps as masses until transfer-matrix or correlation evidence exists.

Do not claim gauge selection until prior and normalization sensitivity have been tested.

Do not claim emergent dimension on a fixed-dimensional lattice.

Do not claim gravity until universal backreaction and causal geometry exist.

Do not claim quantum measurement solution until probabilities and records are derived.

This sequential discipline is essential because the final ToE claim is only as strong as its weakest required bridge.

The research sequence should also permit parallel work.

The exact finite gauge implementation, formal verification, and GenesisBoard extension can proceed together.

Defect analysis and sector selection can share the same software.

Geometry and entropy calculations can be added once the gauge invariants are reliable.

The literature comparison should proceed continuously to distinguish:

known mathematics;

new synthesis;

and original physical prediction.

The program should publish intermediate results individually rather than waiting for a complete ToE.

Possible papers include:

finite groupoid gauge quantization;

defect matter and mass gaps;

dynamical automorphism selection;

measurement as finite homotopy truncation;

relational dimension and geometry;

equivalence entropy and area laws;

and the gravity continuum limit.

The current paper should remain the constructive foundation connecting them.

12.6 Final Conclusion

Modern fundamental physics already depends on equivalence.

Gauge theory identifies field configurations related by local symmetry.

General relativity identifies coordinate presentations of geometry.

Quantum theory identifies vectors differing by physically irrelevant transformations and combines alternative histories through amplitudes.

Statistical mechanics maps many microscopic configurations to one macroscopic meaning.

These theories repeatedly show that mathematical presentation is not identical to physical reality.

Univalent Relational Dynamics elevates this recurring feature into a foundational proposal.

A physical state is a local presentation.

Physical identity is constituted by admissible equivalence.

Persistence is coherent transport across changing contexts.

Gauge symmetry is local automorphism.

Curvature is nontrivial return of transported identity.

Matter is stable obstruction to global trivialization.

Charge is response under self-equivalence.

Mass is the minimum invariant energy required to sustain and propagate a persistent defect.

Quantum amplitude sums physically inequivalent identifications rather than redundant presentations.

Measurement reduces accessible relational structure to stable outcome sectors.

Entropy counts physical multiplicity within coarse equivalence fibers.

Geometry records the cost and organization of identity transport.

Gravity may arise when matter-induced coherence cost changes that geometry universally.

The present paper converts this ontology into a finite constructive framework.

A finite oriented 2-complex supplies local contexts, paths, and faces.

A physical groupoid supplies identity objects, equivalences, and automorphisms.

Edge morphisms transport identity.

Face holonomies measure failed closure.

A conjugacy-invariant action assigns coherence cost.

Matter representations measure agreement under transport.

Gauge orbits and stabilizers define the moduli groupoid.

The quantum partition function sums physical equivalence classes with automorphism weighting.

Defect sectors provide candidates for matter.

Heterogeneous identity sectors provide a mechanism for gauge selection.

Transport costs provide a candidate pregeometry.

This construction contains ordinary lattice gauge theory exactly in the one-object limit.

That recovery is essential because it anchors the new framework in established physics.

But recovery alone is not enough.

The theory becomes physically new only when the enlarged groupoid structure generates a non-inserted result.

The first target should be one of the following:

a stable defect with a calculable invariant gap;

a restricted charge spectrum;

a robust mass ratio;

a dynamically selected automorphism sector;

or an effective geometry not inherited from the lattice.

Such a result would show that equivalence is not merely a language for known redundancy.

It would show that equivalence structure can generate physical organization.

The program remains far from a completed Theory of Everything.

It has not yet derived:

the Standard Model gauge group;

the observed matter representations;

chirality;

three generations;

the Higgs sector;

particle masses;

mixing parameters;

Lorentzian 3+1-dimensional spacetime;

Einstein's equations;

the Born rule;

or cosmological parameters.

These omissions are not minor.

They define the central research agenda.

The framework should therefore be presented neither as a finished ToE nor as mere speculation.

It is a candidate constructive route toward fundamental unification.

Its value lies in reducing a broad metaphysical claim to a sequence of calculations that can fail.

The decisive questions are now explicit.

Can gauge groups emerge as selected automorphism structures?

Can stable defects carry derived charge and mass?

Can a moduli-groupoid quantum sum reproduce known amplitudes?

Can measurement be represented as a physical truncation with correct probabilities?

Can relational transport generate universal Lorentzian geometry?

Can equivalence entropy produce gravitational dynamics?

Can one action connect these results without inserting each sector independently?

The answer to each question must be obtained mathematically and computationally.

A negative answer will narrow or overturn the program.

A positive answer will move it toward a genuine candidate Theory of Everything.

The foundational question of physics is thereby changed.

Instead of beginning with:

What particles, fields, and spacetime points exist?

the theory begins with:

Which local presentations can be coherently identified?

Which transformations preserve physical meaning?

How do those transformations compose?

Where does their composition fail?

What stable structures arise from that failure?

What geometry is generated by the cost of maintaining identity?

The proposed answer is that objects are not primitive occupants of a prior spacetime.

They are persistent solutions to relational coherence.

Gauge fields are not merely mathematical redundancies.

They are the transport architecture of physical identity.

Particles are not necessarily elementary substances.

They may be stable defects in that architecture.

Spacetime is not necessarily the container in which transport occurs.

It may be the large-scale geometry reconstructed from transport itself.

A successful Univalent Relational Dynamics would therefore unify physics by reversing its explanatory order.

It would not place quantum fields and particles into spacetime and then impose symmetry.

It would begin with equivalence, composition, and coherence, and derive fields, particles, and spacetime as effective structures.

The immediate achievement sought here is smaller but indispensable:

to build the first working machine of equivalence-first physics.

If that machine reproduces gauge theory, supports physical defect sectors, quantizes without overcounting, and generates one robust non-inserted result, the project crosses the boundary from philosophical synthesis to constructive fundamental theory.

Only after that boundary is crossed should the larger Theory of Everything claim be judged.

The theory therefore ends not with a declaration of completion but with a precise challenge:

derive the observed structures of physics from the dynamics of admissible equivalence, or identify exactly where the equivalence-first program fails.

13.

I'm assembling Section 13 as a publication-ready numbered bibliography, separating your foundational papers from primary sources on lattice gauge theory, higher gauge theory, HoTT, groupoid quantization, defects, emergent geometry, and thermodynamic gravity.

Thought for 3m 55s

The bibliography below expands the placeholder list into a structured 137-reference foundation. I verified the principal metadata for the related Youvan papers and the central sources on HoTT, higher gauge theory, lattice gauge theory, and thermodynamic gravity. The external foundations are anchored in the official HoTT book, primary higher-gauge literature, Wilson's lattice formulation, and Jacobson's thermodynamic derivation.

13. References

Preprints are identified as such. Journal articles, monographs, and foundational sources are listed in a common numbered sequence so that citations can be added during final manuscript assembly.

13.1 Foundational Papers in the Present Research Program

1. Youvan, D. C. Reality as Equivalence: Univalent Relational Dynamics, Category Theory, Homotopy Type Theory, and an Equivalence-First Foundation for Mathematical Physics. Preprint, June 2026. DOI: 10.13140/RG.2.2.20432.26881.
2. Youvan, D. C. The Cohesive Foundation: Deriving the Laws of Physics from the Axiomatic Adjunction of Homotopy Type Theory. Preprint, April 2026. DOI: 10.13140/RG.2.2.25673.10084.
3. Youvan, D. C. Emergent Gravity: A Comprehensive Analysis of the Thermodynamic Derivation of the Einstein Field Equations. Preprint, June 2026. DOI: 10.13140/RG.2.2.36767.11681.
4. Youvan, D. C. The Noetherian Cosmological Principle: Replacing Observational Uniformity with Fundamental Symmetry. Preprint, June 2025. DOI: 10.13140/RG.2.2.26335.57762.
5. Youvan, D. C. Topos Cosmology Beyond Dark Matter: Gluing Obstructions, Contextual Gravity, and the End of Symmetry as Assumption. Preprint, January 2026. DOI: 10.13140/RG.2.2.11280.67849.
6. Youvan, D. C. Local Truth in Categorical Simulations: GenesisBoard as a Laboratory for Domain-Bounded Coherence. Preprint, August 2025. DOI: 10.13140/RG.2.2.11503.96164.
7. Youvan, D. C. The Minimal Universe-Generating Set: A Foundational Framework for Emergent Reality. Preprint, March 2025. DOI: 10.13140/RG.2.2.33454.22085.
8. Youvan, D. C. Reconstructing Physics from Pure Quantum Computation: A Testable Information-Theoretic Framework for Reality. Preprint, March 2025. DOI: 10.13140/RG.2.2.23414.10560.
9. Youvan, D. C. Category Theory and Homotopy Type Theory, Brick by Brick: A Rigorous LEGO Framework from Morphisms and Universal Properties to Higher Paths and Univalence. Preprint, June 2026. DOI: 10.13140/RG.2.2.10641.19040.

13.2 Category Theory, Infinity-Categories, Homotopy Type Theory, and Univalence

10. Mac Lane, S. Categories for the Working Mathematician. 2nd ed. Springer, 1998.
11. Awodey, S. Category Theory. 2nd ed. Oxford University Press, 2010.
12. Riehl, E. Category Theory in Context. Dover Publications, 2016.
13. Leinster, T. Basic Category Theory. Cambridge University Press, 2014.
14. The Univalent Foundations Program. Homotopy Type Theory: Univalent Foundations of Mathematics. Institute for Advanced Study, 2013.
15. Hofmann, M., and Streicher, T. “The Groupoid Interpretation of Type Theory.” In Twenty-Five Years of Constructive Type Theory, edited by G. Sambin and J. M. Smith, 83–111. Oxford University Press, 1998.
16. Awodey, S., and Warren, M. A. “Homotopy Theoretic Models of Identity Types.” Mathematical Proceedings of the Cambridge Philosophical Society 146 (2009): 45–55. DOI: 10.1017/S0305004108001783.
17. Kapulkin, K., Lumsdaine, P. L., and Voevodsky, V. “The Simplicial Model of Univalent Foundations (after Voevodsky).” Journal of the European Mathematical Society 23 (2021): 2071–2126.
18. Rijke, E., Shulman, M., and Spitters, B. “Modalities in Homotopy Type Theory.” Logical Methods in Computer Science 16, no. 1 (2020), Article 2. DOI: 10.23638/LMCS-16(1:2)2020.
19. Shulman, M. “Univalence for Inverse Diagrams and Homotopy Canonicity.” Mathematical Structures in Computer Science 25 (2015): 1203–1277.
20. Lurie, J. Higher Topos Theory. Princeton University Press, 2009.
21. Riehl, E., and Verity, D. Elements of Infinity-Category Theory. Cambridge University Press, 2022.
22. Leinster, T. Higher Operads, Higher Categories. Cambridge University Press, 2004.
23. Baez, J. C., and Dolan, J. “Higher-Dimensional Algebra III: n-Categories and the Algebra of Opetopes.” Advances in Mathematics 135 (1998): 145–206. DOI: 10.1006/aima.1997.1695.
24. Rezk, C. “A Model for the Homotopy Theory of Homotopy Theory.” Transactions of the American Mathematical Society 353 (2001): 973–1007.

25. Lawvere, F. W. “Axiomatic Cohesion.” Theory and Applications of Categories 19 (2007): 41–49.

13.3 Cohesive Homotopy Theory, Higher Gauge Theory, and Differential Geometry

26. Schreiber, U., and Shulman, M. “Quantum Gauge Field Theory in Cohesive Homotopy Type Theory.” Electronic Proceedings in Theoretical Computer Science 158 (2014): 109–126. DOI: 10.4204/EPTCS.158.8.

27. Schreiber, U. Differential Cohomology in a Cohesive Infinity-Topos. arXiv:1310.7930, 2013.

28. Schreiber, U. Classical Field Theory via Cohesive Homotopy Types. arXiv:1311.1172, 2013.

29. Baez, J. C., and Schreiber, U. “Higher Gauge Theory.” In Categories in Algebra, Geometry and Mathematical Physics, Contemporary Mathematics 431, 7–30. American Mathematical Society, 2007.

30. Baez, J. C., and Schreiber, U. Higher Gauge Theory: 2-Connections on 2-Bundles. arXiv/0412325, 2004.

31. Baez, J. C., and Huerta, J. “An Invitation to Higher Gauge Theory.” General Relativity and Gravitation 43 (2011): 2335–2392. DOI: 10.1007/s10714-010-1070-9.

32. Baez, J. C., and Lauda, A. D. “Higher-Dimensional Algebra V: 2-Groups.” Theory and Applications of Categories 12 (2004): 423–491.

33. Breen, L., and Messing, W. “Differential Geometry of Gerbes.” Advances in Mathematics 198 (2005): 732–846.

34. Murray, M. K. “Bundle Gerbes.” Journal of the London Mathematical Society 54 (1996): 403–416. DOI: 10.1112/jlms/54.2.403.

35. Sati, H., Schreiber, U., and Stasheff, J. “Twisted Differential String and Fivebrane Structures.” Communications in Mathematical Physics 315 (2012): 169–213. DOI: 10.1007/s00220-012-1510-3.

36. Brylinski, J.-L. Loop Spaces, Characteristic Classes and Geometric Quantization. Birkhäuser, 1993.

37. Freed, D. S. “Classical Chern–Simons Theory, Part 1.” Advances in Mathematics 113 (1995): 237–303.

38. Baez, J. C., and Muniain, J. P. Gauge Fields, Knots and Gravity. World Scientific, 1994.

39. Kobayashi, S., and Nomizu, K. Foundations of Differential Geometry. Vols. 1–2. Wiley, 1963–1969.

40. Nakahara, M. Geometry, Topology and Physics. 2nd ed. Institute of Physics Publishing, 2003.

13.4 Gauge Theory, Lattice Gauge Theory, and Groupoid Quantization

41. Weyl, H. “Gravitation und Elektrizität.” Sitzungsberichte der Königlich Preussischen Akademie der Wissenschaften zu Berlin (1918): 465–480.

42. Weyl, H. “Elektron und Gravitation. I.” Zeitschrift für Physik 56 (1929): 330–352.

43. Yang, C. N., and Mills, R. L. “Conservation of Isotopic Spin and Isotopic Gauge Invariance.” Physical Review 96 (1954): 191–195. DOI: 10.1103/PhysRev.96.191.

44. Utiyama, R. “Invariant Theoretical Interpretation of Interaction.” Physical Review 101 (1956): 1597–1607. DOI: 10.1103/PhysRev.101.1597.

45. Wilson, K. G. “Confinement of Quarks.” Physical Review D 10 (1974): 2445–2459. DOI: 10.1103/PhysRevD.10.2445.

46. Kogut, J., and Susskind, L. “Hamiltonian Formulation of Wilson’s Lattice Gauge Theories.” Physical Review D 11 (1975): 395–408. DOI: 10.1103/PhysRevD.11.395.

47. Kogut, J. B. “An Introduction to Lattice Gauge Theory and Spin Systems.” Reviews of Modern Physics 51 (1979): 659–713. DOI: 10.1103/RevModPhys.51.659.

48. Wegner, F. J. “Duality in Generalized Ising Models and Phase Transitions without Local Order Parameters.” Journal of Mathematical Physics 12 (1971): 2259–2272.

49. Fradkin, E., and Shenker, S. H. “Phase Diagrams of Lattice Gauge Theories with Higgs Fields.” Physical Review D 19 (1979): 3682–3697. DOI: 10.1103/PhysRevD.19.3682.

50. Elitzur, S. “Impossibility of Spontaneously Breaking Local Symmetries.” Physical Review D 12 (1975): 3978–3982. DOI: 10.1103/PhysRevD.12.3978.

51. Creutz, M. Quarks, Gluons and Lattices. Cambridge University Press, 1983.

52. Montvay, I., and Münster, G. Quantum Fields on a Lattice. Cambridge University Press, 1994.

53. Rothe, H. J. Lattice Gauge Theories: An Introduction. 4th ed. World Scientific, 2012.

54. Gattringer, C., and Lang, C. B. Quantum Chromodynamics on the Lattice. Springer, 2010.

55. Zohar, E., Cirac, J. I., and Reznik, B. “Quantum Simulations of Lattice Gauge Theories Using Ultracold Atoms in Optical Lattices.” Reports on Progress in Physics 79 (2016): 014401.

56. Dijkgraaf, R., and Witten, E. “Topological Gauge Theories and Group Cohomology.” Communications in Mathematical Physics 129 (1990): 393–429.

57. Freed, D. S., and Quinn, F. “Chern–Simons Theory with Finite Gauge Group.” Communications in Mathematical Physics 156 (1993): 435–472.

58. Freed, D. S., Hopkins, M. J., Lurie, J., and Teleman, C. “Topological Quantum Field Theories from Compact Lie Groups.” In A Celebration of the Mathematical Legacy of Raoul Bott, CRM Proceedings and Lecture Notes 50, 367–403. American Mathematical Society, 2010.

59. Baez, J. C., Hoffnung, A. E., and Walker, C. D. “Higher-Dimensional Algebra VII: Groupoidification.” Advances in Theoretical and Mathematical Physics 15 (2011): 1373–1410.

60. Baez, J. C., and Dolan, J. “From Finite Sets to Feynman Diagrams.” In Mathematics Unlimited—2001 and Beyond, edited by B. Engquist and W. Schmid, 29–50. Springer, 2001.

61. Weinstein, A. “The Volume of a Differentiable Stack.” Letters in Mathematical Physics 90 (2009): 353–371.

62. Atiyah, M. F. “Topological Quantum Field Theories.” Publications Mathématiques de l’IHÉS 68 (1988): 175–186.

63. Kitaev, A. Y. “Fault-Tolerant Quantum Computation by Anyons.” Annals of Physics 303 (2003): 2–30. DOI: 10.1016/S0003-4916(02)00018-0.

13.5 Topological Defects, Solitons, Charge, and Mass Gaps

64. Mermin, N. D. “The Topological Theory of Defects in Ordered Media.” Reviews of Modern Physics 51 (1979): 591–648. DOI: 10.1103/RevModPhys.51.591.

65. Kibble, T. W. B. “Topology of Cosmic Domains and Strings.” Journal of Physics A: Mathematical and General 9 (1976): 1387–1398.

66. Nielsen, H. B., and Olesen, P. “Vortex-Line Models for Dual Strings.” Nuclear Physics B 61 (1973): 45–61.

67. ’t Hooft, G. “Magnetic Monopoles in Unified Gauge Theories.” Nuclear Physics B 79 (1974): 276–284.

68. Polyakov, A. M. “Particle Spectrum in Quantum Field Theory.” JETP Letters 20 (1974): 194–195.

69. Skyrme, T. H. R. “A Non-Linear Field Theory.” Proceedings of the Royal Society A 260 (1961): 127–138.

70. Rajaraman, R. Solitons and Instantons. North-Holland, 1982.

71. Vilenkin, A., and Shellard, E. P. S. Cosmic Strings and Other Topological Defects. Cambridge University Press, 1994.

72. Coleman, S. Aspects of Symmetry. Cambridge University Press, 1985.

73. Tong, D. TASI Lectures on Solitons. arXiv/0509216, 2005.

74. Atiyah, M. F., and Manton, N. S. “Skyrmions from Instantons.” Physics Letters B 222 (1989): 438–442.

75. Levin, M., and Wen, X.-G. “String-Net Condensation: A Physical Mechanism for Topological Phases.” Physical Review B 71 (2005): 045110.

76. Wen, X.-G. Quantum Field Theory of Many-Body Systems. Oxford University Press, 2004.

77. Morningstar, C. J., and Peardon, M. “Glueball Spectrum from an Anisotropic Lattice Study.” Physical Review D 60 (1999): 034509.

13.6 Quantum Amplitudes, Decoherence, and Measurement

78. Feynman, R. P., and Hibbs, A. R. Quantum Mechanics and Path Integrals. McGraw-Hill, 1965.

79. Schulman, L. S. Techniques and Applications of Path Integration. Dover Publications, 2005.

80. Nielsen, M. A., and Chuang, I. L. Quantum Computation and Quantum Information. 10th anniversary ed. Cambridge University Press, 2010.

81. Zurek, W. H. “Decoherence, Einselection, and the Quantum Origins of the Classical.” Reviews of Modern Physics 75 (2003): 715–775. DOI: 10.1103/RevModPhys.75.715.

82. Schlosshauer, M. “Decoherence, the Measurement Problem, and Interpretations of Quantum Mechanics.” Reviews of Modern Physics 76 (2005): 1267–1305. DOI: 10.1103/RevModPhys.76.1267.

83. Joos, E., Zeh, H. D., Kiefer, C., Giulini, D., Kupsch, J., and Stamatescu, I.-O. Decoherence and the Appearance of a Classical World in Quantum Theory. 2nd ed. Springer, 2003.

84. Griffiths, R. B. “Consistent Histories and the Interpretation of Quantum Mechanics.” Journal of Statistical Physics 36 (1984): 219–272.

85. Gell-Mann, M., and Hartle, J. B. “Quantum Mechanics in the Light of Quantum Cosmology.” In Complexity, Entropy, and the Physics of Information, edited by W. H. Zurek, 425–458. Addison-Wesley, 1990.

86. Gleason, A. M. “Measures on the Closed Subspaces of a Hilbert Space.” Journal of Mathematics and Mechanics 6 (1957): 885–893.

87. Lüders, G. “Über die Zustandsänderung durch den Messprozess.” Annalen der Physik 443 (1951): 322–328.

88. Stinespring, W. F. “Positive Functions on C*-Algebras.” Proceedings of the American Mathematical Society 6 (1955): 211–216.

89. Kraus, K. States, Effects, and Operations. Springer, 1983.

90. Everett, H. “Relative State Formulation of Quantum Mechanics.” Reviews of Modern Physics 29 (1957): 454–462.

13.7 Emergent Geometry, Entanglement, Entropy, and Gravity

91. Regge, T. “General Relativity without Coordinates.” Il Nuovo Cimento 19 (1961): 558–571.

92. Bombelli, L., Lee, J., Meyer, D., and Sorkin, R. D. “Space-Time as a Causal Set.” Physical Review Letters 59 (1987): 521–524.

93. Ambjørn, J., Jurkiewicz, J., and Loll, R. “Spectral Dimension of the Universe.” Physical Review Letters 95 (2005): 171301.

94. Ollivier, Y. “Ricci Curvature of Markov Chains on Metric Spaces.” Journal of Functional Analysis 256 (2009): 810–864.

95. Forman, R. “Bochner’s Method for Cell Complexes and Combinatorial Ricci Curvature.” Discrete and Computational Geometry 29 (2003): 323–374. DOI: 10.1007/s00454-002-0743-x.

96. Van Raamsdonk, M. “Building Up Spacetime with Quantum Entanglement.” General Relativity and Gravitation 42 (2010): 2323–2329.

97. Swingle, B. “Entanglement Renormalization and Holography.” Physical Review D 86 (2012): 065007.

98. Ryu, S., and Takayanagi, T. “Holographic Derivation of Entanglement Entropy from the Anti-de Sitter Space/Conformal Field Theory Correspondence.” Physical Review Letters 96 (2006): 181602.

99. Pastawski, F., Yoshida, B., Harlow, D., and Preskill, J. “Holographic Quantum Error-Correcting Codes: Toy Models for the Bulk/Boundary Correspondence.” Journal of High Energy Physics 06 (2015): 149.

100. Almheiri, A., Dong, X., and Harlow, D. “Bulk Locality and Quantum Error Correction in AdS/CFT.” Journal of High Energy Physics 04 (2015): 163.

101. Cao, C., Carroll, S. M., and Michalakis, S. “Space from Hilbert Space: Recovering Geometry from Bulk Entanglement.” Physical Review D 95 (2017): 024031.

102. Bekenstein, J. D. “Black Holes and Entropy.” Physical Review D 7 (1973): 2333–2346.

103. Hawking, S. W. “Particle Creation by Black Holes.” Communications in Mathematical Physics 43 (1975): 199–220.

104. Unruh, W. G. “Notes on Black-Hole Evaporation.” Physical Review D 14 (1976): 870–892.

105. Jacobson, T. “Thermodynamics of Spacetime: The Einstein Equation of State.” Physical Review Letters 75 (1995): 1260–1263. DOI: 10.1103/PhysRevLett.75.1260.

106. Eling, C., Guedens, R., and Jacobson, T. “Nonequilibrium Thermodynamics of Spacetime.” Physical Review Letters 96 (2006): 121301.

107. Padmanabhan, T. “Thermodynamical Aspects of Gravity: New Insights.” Reports on Progress in Physics 73 (2010): 046901.

108. Verlinde, E. P. “On the Origin of Gravity and the Laws of Newton.” Journal of High Energy Physics 04 (2011): 029.

109. Jacobson, T. “Entanglement Equilibrium and the Einstein Equation.” Physical Review Letters 116 (2016): 201101.

110. Lashkari, N., McDermott, M. B., and Van Raamsdonk, M. “Gravitational Dynamics from Entanglement Thermodynamics.” Journal of High Energy Physics 04 (2014): 195.

111. Faulkner, T., Guica, M., Hartman, T., Myers, R. C., and Van Raamsdonk, M. “Gravitation from Entanglement in Holographic CFTs.” Journal of High Energy Physics 03 (2014): 051.

112. Wald, R. M. “Black Hole Entropy Is the Noether Charge.” Physical Review D 48 (1993): R3427–R3431.

113. Iyer, V., and Wald, R. M. “Some Properties of Noether Charge and a Proposal for Dynamical Black Hole Entropy.” Physical Review D 50 (1994): 846–864.

114. Bousso, R. “The Holographic Principle.” Reviews of Modern Physics 74 (2002): 825–874.

115. Carlip, S. “Dimension and Dimensional Reduction in Quantum Gravity.” Classical and Quantum Gravity 34 (2017): 193001.

116. Wald, R. M. General Relativity. University of Chicago Press, 1984.

13.8 Noether Structure, Standard-Model Benchmarks, Computation, and Formal Verification

117. Noether, E. “Invariante Variationsprobleme.” Nachrichten von der Gesellschaft der Wissenschaften zu Göttingen, Mathematisch-Physikalische Klasse (1918): 235–257.

118. Kosmann-Schwarzbach, Y. The Noether Theorems: Invariance and Conservation Laws in the Twentieth Century. Springer, 2011.

119. Lee, J., and Wald, R. M. “Local Symmetries and Constraints.” Journal of Mathematical Physics 31 (1990): 725–743.

120. Higgs, P. W. “Broken Symmetries and the Masses of Gauge Bosons.” Physical Review Letters 13 (1964): 508–509.

121. Englert, F., and Brout, R. “Broken Symmetry and the Mass of Gauge Vector Mesons.” Physical Review Letters 13 (1964): 321–323.

122. Weinberg, S. “A Model of Leptons.” Physical Review Letters 19 (1967): 1264–1266.

123. Gross, D. J., and Wilczek, F. “Ultraviolet Behavior of Non-Abelian Gauge Theories.” Physical Review Letters 30 (1973): 1343–1346.

124. Politzer, H. D. “Reliable Perturbative Results for Strong Interactions?” Physical Review Letters 30 (1973): 1346–1349.

125. Nielsen, H. B., and Ninomiya, M. “No Go Theorem for Regularizing Chiral Fermions.” Physics Letters B 105 (1981): 219–223.

126. Ginsparg, P. H., and Wilson, K. G. “A Remnant of Chiral Symmetry on the Lattice.” Physical Review D 25 (1982): 2649–2657.

127. Metropolis, N., Rosenbluth, A. W., Rosenbluth, M. N., Teller, A. H., and Teller, E. “Equation of State Calculations by Fast Computing Machines.” Journal of Chemical Physics 21 (1953): 1087–1092.

128. Hastings, W. K. “Monte Carlo Sampling Methods Using Markov Chains and Their Applications.” Biometrika 57 (1970): 97–109.

129. Swendsen, R. H., and Wang, J.-S. “Nonuniversal Critical Dynamics in Monte Carlo Simulations.” Physical Review Letters 58 (1987): 86–88.

130. Wolff, U. “Collective Monte Carlo Updating for Spin Systems.” Physical Review Letters 62 (1989): 361–364.

131. Hukushima, K., and Nemoto, K. “Exchange Monte Carlo Method and Application to Spin Glass Simulations.” Journal of the Physical Society of Japan 65 (1996): 1604–1608.

132. Ferrenberg, A. M., and Swendsen, R. H. “Optimized Monte Carlo Data Analysis.” Physical Review Letters 63 (1989): 1195–1198.

133. Newman, M. E. J., and Barkema, G. T. Monte Carlo Methods in Statistical Physics. Oxford University Press, 1999.

134. The GAP Group. GAP—Groups, Algorithms, and Programming. Computer algebra system and reference manual.

135. Bezem, M., Coquand, T., and Huber, S. “A Model of Type Theory in Cubical Sets.” In 19th International Conference on Types for Proofs and Programs, LIPIcs 26, 107–128. Schloss Dagstuhl, 2014.

136. Cohen, C., Coquand, T., Huber, S., and Mörtberg, A. “Cubical Type Theory: A Constructive Interpretation of the Univalence Axiom.” In 21st International Conference on Types for Proofs and Programs, LIPIcs 69, Article 5. Schloss Dagstuhl, 2018.

137. Vezzosi, A., Mörtberg, A., and Abel, A. “Cubical Agda: A Dependently Typed Programming Language with Univalence and Higher Inductive Types.” Journal of Functional Programming 31 (2021): e11.

The author has published over 5,600 AI-assisted papers at: https://www.researchgate.net/profile/Douglas-Youvan/research . Google Scholar has indexed these preprints, and if you want a particular reference, the AI mode of a Google search (Gemini) has efficiently catalogued these preprints.